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ABSTRACT

This guide for teachers in the Houston schools provides plans for teaching over 80 objectives in the mathematics program for grade 7. For each lesson, the objectives are specified, teaching activities are given, and supplementary materials are listed. Practice exercises, answer keys, and mathematical background comprise the bulk of the guide, and a glossary is given at the end. (MNS)

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PRE-ALGEBRA

GRADE 7A-B

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HOUSTON INDEPENDENT SCHOOL DISTRICT

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Grade 7A - B

Curriculum Bulletin 82CBM 34

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Abbreviations used:

AWA — Adventures with Arithmetic
MS — Mathimagination Series
MT — Mathematics Teacher - journal
MAH — Math Activities Handbook

7H — Additional Material for Individualizing (Scott Foresman)
7L — Additional Material for Individualizing (Scott Foresman)
8H — Additional Material for Individualizing (Scott Foresman)
8L — Additional Material for Individualizing (Scott Foresman)

Practice 7 — Practice and Activities Workbook (Scott Foresman)
Practice 8 — Practice and Activities Workbook (Scott Foresman)

Skillmaster 7 — Skillmaster Book (Silver Burdett)
Skillmaster 8 — Skillmaster Book (Silver Burdett)

MRC — Math Resource Center Materials
PAWP — Pre-Alegbra with Pizzaz
AM/DGS — Amusements in Mathematics and Developing Geometry Series III

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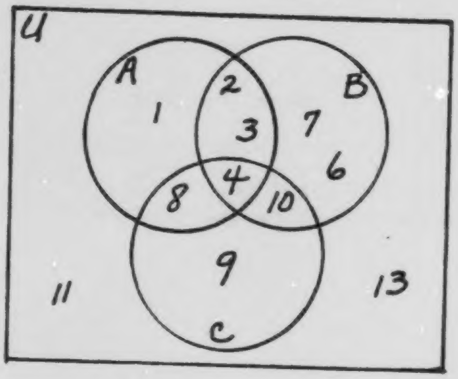
Grade 7a

Mathematics Objective 1: Identify a relationship between two sets
2: Choose the correct Interpretation of a venn diagram

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
	In Guide: Sets: . A brief survey Worksheet - Sets p. 1 p. 2
	MRC Materials AWA (IV) p. 42-43 AWA (IV) p. 44,45 MS-C7, C8, C9 Eureka No. 1

Worksheet – Sets

Using this venn diagram give the elements of the following sets in roster form:



1. $U =$
2. $A =$
3. $B =$
4. $C =$
5. $A \cap C =$
6. $A \cup B =$
7. $C \cap B =$
8. $(B \cap A) \cap \phi =$
9. $A \cup (B \cup C) =$
10. $A \cap (B \cup C) =$

Identify the following statements as true or false.

11. $A \subset B$ ________
12. $C \subset U$ ________
13. $4 \in A$ ________
14. $A \cup B = B \cup A$ ________
15. $A \subset A$ ________
16. $A \cap B \cap C = 4$ ________

Identify these sets by listing their members.

17. The set of whole numbers larger than five and less than nine. ________________.
18. The set of numbers between 10 and 20 that are divisible by six ________________.
19. The days of the week whose name begins with T. ________________
20. The letters that make up the word Mississippi ________________
21. Months of the year that have more days than the previous month. ________________

Give the correct symbol for each of the following.

22. set ________
23. equals ________
24. is a member of ________
25. is a subset ________
26. Empty or null set ________
27. union ________
28. intersection ________

Sets: A Brief Survey

What is a set? Everyday we consider collections of things. These collections could be called sets. We might talk about a collections or a set of stamps, or a group or set of five girls. In mathematics we might talk about the set of digits used in the decimal system of numerals, or a set of problems to be done.

We will use the word "set" to refer to any collection or group of things. Each separate thing in a set is an element or member of the set.

In writing about a set we can use braces – { } – to enclose the names of the set elements. We use commas to separate the names of the elements. For example, the set of our first two Presidents of the united States would be {George Washington, John Adams} .

We use capital letters to name sets. For example we might write, "Let A = {3, 5, 7, 9} ." The symbol '=' indicates that A and {3, 5, 7, 9} refer to the same set.

Equal sets: Two sets are equal if they contain exactly the same elements. For example: {0, 2, 4, 6, 8} = {all even numbers from zero through 8} . Thus {0, 2, 4, 6, 8} can be read as "the set whose elements are 0, 2, 4, 6, 8" or "the set of all even numbers zero through eight".

Equivalent sets: Equivalent sets have the same number of elements in each set. Thus P = {a, b, c} and R = {1, 2, 3} are equivalent sets.

Empty or null sets: Could we have a set that does not contain any elements? How would you list the elements of the set of two - headed students in your class? The set that has no elements is called an empty set or a null set. The symbol for this set is 'ϕ' which is the Greek letter "phi". Note that the symbol ϕ is **not** enclosed in set braces.

When expressing the set of even numbers 2 through 200 by roster, we can use three dots and omit many numerals. Example: R = {2, 4, 6, . . . , 200} .

The symbol for element or member is $\in$, thus 4 $\in$ {2, 4, 6, , 200} , four is a member of the set of even numbers 2 through 200.

Finite set: If a natural number can express the number of elements in a set, then the set is a finite set.

Infinite set: If a natural number cannot be used to express the number of elements in a set, then the set is an infinite set. We can express an infinite set N = {0, 2, 4, 6, . . .} . The series of dots means "and so on" in the same pattern indefinitely. It could also be written N = {all even whole numbers}

Union of sets: putting together sets is forming the union of those sets. The symbol for the operation of forming the union of two sets is " ∪" (cup) set B = {1, 2, 3, 4} set N = {3, 4, 5, 6} B ∪ N = {1, 2, 3, 4, 5, 6} note that we do not write the elements 3 and 4 twice since they appear only once in each set.

Intersection: The intersection of any two sets A and B is the set containing all the elements that are common to both set A and set B. The symbol for intersection is '∩' (cap) For example: M = {2, 3, 5, 7} N = {1, 3, 5, 7} , P = {2, 4, 6, 8 M ∩N = 3, 5, 7 M∩P = 2 N∩P = ϕ Sets that have no intersection are sometimes called disjoint sets.

Subsets: The set A is a subset of set B, if all elements of a are also elements of B'. A= {1, 3, 5} B= {1, 3, 5, . . .} ∵ A⊂ B. Every set is a subset of itself. The empty set is a subset of every set. Each member is a subset of the set and each possible combination of members is a subset of the set.

To specify a set, you must identify its members, or elements. One way to specify a set is by roster or listing the members. Another way to specify a set is by word **description.**

roster	description
{1, 2, 3, 4, 5, ...}	{natural numbers}
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9,}	{digits}
{x, y, z,}	{the last three letters of the alphabet}

When the set is referred to often and has too many members to let conveniently each time it is mentioned, a capitol letter is assigned to the set and only the capitol letter is referred to.

V = {1, 2, 3, . . .} = natural numbers
W = {0, 1, 2, 3, . . .} = whole numbers

Complete the following when

$A = \{4, 8, 12, 16\}$ $B = \{2, 4, 6, 8\}$ $C = \{2, 3, 6, 7\}$

29. $B \cup C =$ __________

30. $A \cup C =$ __________

31. $B \cap C =$ __________

32. $A \cap C =$ __________

33. Is $A \cap B = B \cap A$? __________

If $A = \{0, 2, 4, 6, 8, \ldots\}$

$B = \{1, 3, 5, 7, 9 \ldots\}$

$C = \{4, 8, 12\}$

$D = \{7, 14, 21, 28\}$

$E = \{5, 6, 7, 9, 10, 11\}$

Then 34. $A \cup B = \{ \qquad \}$

35. $D \cap E = \{ \qquad \}$

36. $A \cap B = \{ \qquad \}$

37 $B \cap E = \{ \qquad \}$

38. $(C \cup E) \cap B = \{ \qquad \}$

39. If $M \cup N = \phi$ what can be said about sets M and N. ______________________________.

*Write a roster description of each set

40. {counting numbers less than 6} ______________________________

41. {each whole numbers between 5 and 13} ______________________________

Write a word description of each set

42. $\{1, 3, 5, 7, \ldots\}$ ______________________________

43. $\{0, 1, 2, 3, 4\}$ ______________________________

44. $6, 8, 10, \ldots, 22\}$ ______________________________

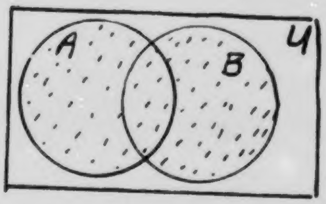
Answer Sheet

Sets

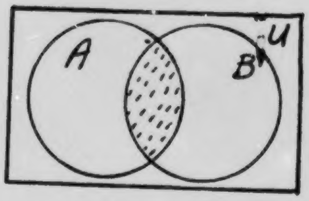
1. $\{1,2,3,4,6,7,8,9,10,11,13\}$
2. $\{1,2,3,4,8\}$
3. $\{2,3,4,6,7,10\}$
4. $\{4,8,9,10\}$
5. $\{4,8\}$
6. $\{1, 2, 3, 4, 6, 7, 8, 10\}$
7. $\{4,10\}$
8. $\{2,3,4\}$
9. $\{1,2,3,4,6,7,8,9,10\}$,
10. $\{2,3,4,8\}$
11. False
12. True
13. True
14. True
15. True
16. True
17. W = $\{6,7,8\}$
18. N = $\{12,18\}$
19. T = {Tuesday, Thursday}
20. M = $\{m,i,s,p\}$
21. Y = {March, May, July, October, December}
22. $\{\ \}$
23. =
24. $\in$
25. $\subset$
26. ϕ
27. $\cup$
28. $\cap$
29. $\{2,3,4,6,7,8,\}$
30. $\{2, 3, 4, 6, 7, 8, 12, 16\}$
31. $\{2,6\}$
32. ϕ
33. yes
34. $\{0,1,2,3,4,5,\ldots\}$
35. $\{7\}$
36. ϕ
37. $\{5,7,9,11\}$
38. $\{5, 7, 9, 11\}$
39. They are empty sets
40. $\{1, 2, 3, 4, 5\}$
41. $\{6, 8, 10, 12\}$
42. odd natural numbers
43. whole numbers less than five.
44. even numbers between 5 and 23.

Venn Diagrams

Diagrams used to picture set relationships are called Venn diagrams in honor of the English mathematician John Venn, who was one of the first to use them for this purpose.

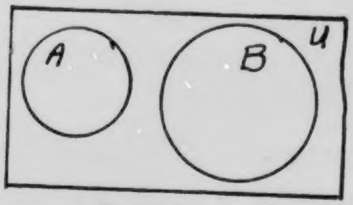


$A \cup B$

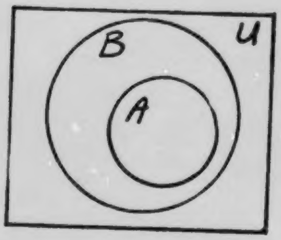


$A \cap B$

The interior of the rectangle represents the universal set, U, that is, the set that contains all the numbers in our discussion.



$A \cap B = \emptyset$



$A \subset B$

If there are no members in $A \cap B$, then we write $A \cap B = \emptyset$, where $\emptyset$ denotes the null set or empty set. The sets A and B in this case are said to be disjoint.

Venn Diagrams
Worksheet

Let $A = \{3, 6, 9\}$, $B = \{6, 12, 18\}$, $C = \{15\}$, $D = \{2, 4, 6, 8\}$, and $E = \{4, 6\}$.

For each of the following, draw a Venn diagram and list the members of the resulting set:

① $A \cap B$

② $A \cup B$

③ $A \cap C$

④ $A \cup C$

⑤ $B \cap C$

⑥ $B \cup C$

⑦ $E \subset D$ or $E \cup D$

⑧ $D \cap E$

Make each true:

⑨ 9 _____ A

⑩ 18 _____ A

⑪ {4,6} _____ D

⑫ 2 _____ D

⑬ {6, 12, 18} _____ B

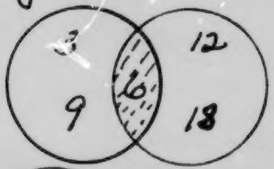
⑭ 3 _____ B

⑮ {2, 4, 8} _____ B

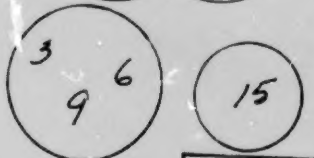
Answers

Venn Diagrams

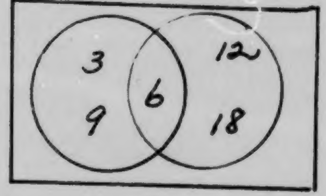
① {6}



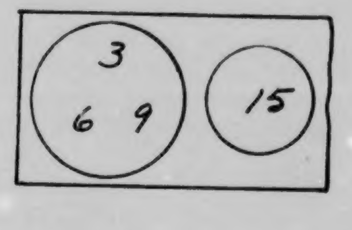
② ∅



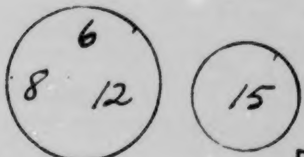
③ {3, 6, 9, 12, 18}



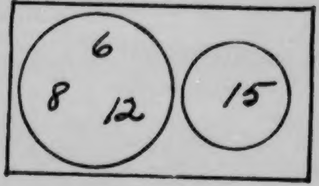
④ {3, 6, 9, 15}



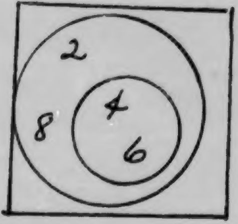
⑤ ∅



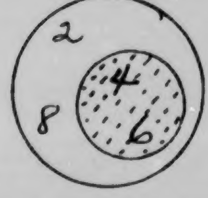
⑥ {6, 12, 15, 18}



⑦ {2, 4, 6, 8}



⑧ {4, 6}



⑨ ∈

⑩ ∉

⑪ ⊂

⑫ ∈

⑬ ⊂

⑭ ∉

⑮ ⊄

Mathematics Objective

- 3M Read and write whole numbers
- 3M Interpret place value
- 3M Round whole numbers ot hte nearest tens; hundreds; thousands

Teaching Techniques and Introductory Activities

Place Value
For teaching suggestions see page 26, 9 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Rounding and estimating
For teaching suggestions see page 8-13 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Place Value
For teaching suggestions see page 9 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Rounding and estimating
For teaching suggestions see page 19-20 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Have the students read aloud as many numbers representing the numerals in the exercises as is practical.

Supplementary Activities and Resources

Practice and Activities Workbook 7 p. 3, 16
7-Additional Materials for Individualizing-L-4

8 practice p. 2
8 L-2

Place value
For teaching suggestions see page 68-69 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Rounding
For teaching suggestions see page 6-7 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

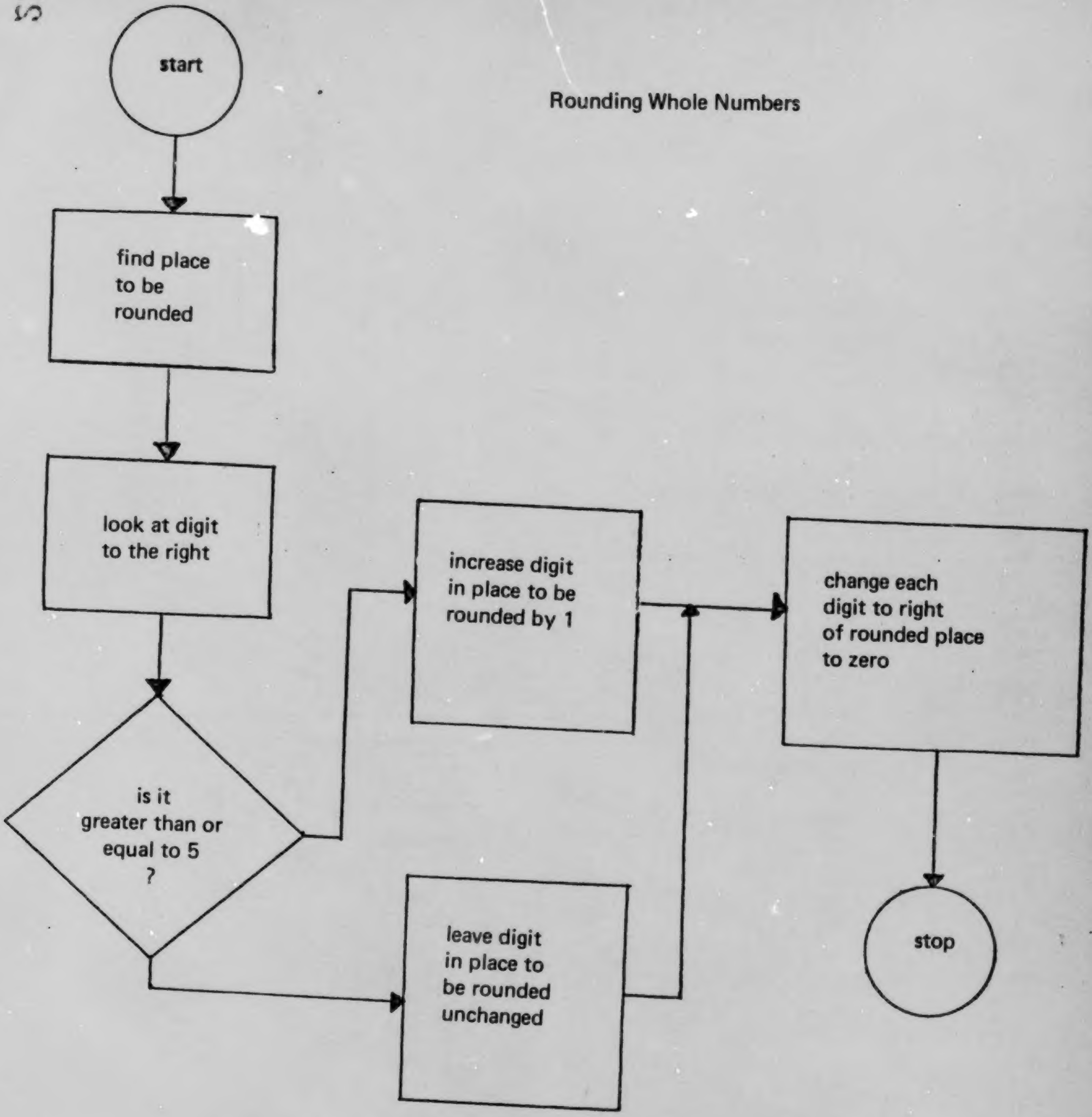
MRC.
MS - C-10, C-11
AS - II p. 25

Place Value
For teaching suggestions see page 8 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

Rounding
For teaching suggestions see page 8-10 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

In Guide: Flow chart: Rounding whole numbers
Chart: About positive numbers
Worksheet: Reading and writing numbers

Rounding Whole Numbers


start
find place to be rounded
look at digit to the right
is it greater than or equal to 5 ?
increase digit in place to be rounded by 1
leave digit in place to be rounded unchanged
change each digit to right of rounded place to zero
stop

from *Mathematics for Mastery* (Silver Burdett) p. 9

About Positive Numbers...

NUMBERS GREATER THAN ONE

10^x	Common Name	Prefix	Abbrev.
10^0	ONE		
10^1	TEN	DEKA	da
10^2	1-HUNDRED	HECTO	h
10^3	1-THOUSAND	KILO	k
10^6	1-MILLION	MEGA	M
10^9	1-BILLION	*GIGA	G
10^{12}	1-TRILLION	TERA	T
10^{15}	1-QUADRILLION		
10^{18}	1-QUINTILLION		
10^{21}	1-SEXTILLION		
10^{24}	1-SEPTILLION		
10^{27}	1-OCTILLION		
10^{30}	1-NONILLION		
10^{33}	1-DECILLION		
10^{36}	1-UNDECILLION		

NUMBERS LESS THAN ONE

10^x	Common Name	Prefix	Abbre
10^{-1}	1-TENTH	DECI	d
10^{-2}	1-HUNDREDTH	CENTI	c
10^{-3}	1-THOUSANDTH	MILLI	m
10^{-4}	1-TEN THOUSANDTH		
10^{-5}	1-HUNDRED THOUSANDTH		
10^{-6}	1-MILLIONTH	MICRO	μ
10^{-7}	1-TEN MILLIONTH		
10^{-8}	1-HUNDRED MILLIONTH		
10^{-9}	1-BILLIONTH	NANO	n
10^{-10}	1-TEN BILLIONTH		
10^{-11}	1-HUNDRED BILLIONTH		
10^{-12}	1-TRILLIONTH	PICO	p
10^{-13}	1-TEN TRILLIONTH		
10^{-14}	1-HUNDRED TRILLIONTH		
10^{-15}	1-QUADRILLIONTH	FEMTO	f

10^{39}	1-DUODECILLION
10^{42}	1-TREDECILLION
10^{45}	1-QUATTOURDECILLION
10^{48}	1-QUINDECILLION
10^{51}	1-SEXDECILLION
10^{54}	1-SEPTENDECILLION
10^{57}	1-OCTODECILLION
10^{60}	1-NOVEMDECILLION
10^{63}	1-VIGINTILLION
10^{98}	1-TRIGINTILLION
10^{100}	1-GOOGOL (10-Duotrigintillion)
10^{123}	1-QUADRAGINTILLION
10^{153}	1-QUINQUAGINTILLION
10^{183}	1-SEXAGINTILLION
10^{213}	1-SEPTAGINTILLION
10^{243}	1-OCTOGINTILLION
10^{273}	1-NONAGINTILLION
10^{303}	1-CENTILLION
10^{GOOGOL}	1-GOOGOLPLEX $10^{(10^{100})}$

10^{-16}	1-TEN QUADRILLIONTH		
10^{-17}	1-HUNDRED QUADRILLIONTH		
10^{-18}	1-QUINTILLIONTH	ATTO	a
10^{-19}	1-TEN QUINTILLIONTH		
10^{-20}	1-HUNDRED QUINTILLIONTH		
10^{-21}	1-SEXTILLIONTH		
10^{-22}	1-TEN SEXTILLIONTH		
10^{-23}	1-HUNDRED SEXTILLIONTH		
10^{-24}	1-SEPTILLIONTH		

Scientific names and abbreviations are the official ones adopted by the International Committee of Weights and Measures.

*The average human life span is only slightly more than two Giga seconds.

Litton MONROE
the calculator company

Write the correct numeral for each number named.

① thirty-two million, four hundred ninety-six thousand, one hundred fifty-three ______

② one billion ______

③ six hundred million, seven hundred eighty-three thousand, nine hundred ______

④ one trillion ______

⑤ six million, six thousand, six ______

⑥ six million, four hundred twenty-three thousand, seven hundred thirty-five ______

⑦ twenty-seven billion, five hundred eighty-four thousand ______

⑧ thirty-eight million, forty-three thousand, two hundred five ______

⑨ three billion, seven million, two hundred thirty thousand, nine ______

⑩ two and thirty-five hundredths ______

⑪ one thousand, three ______

⑫ four hundred eighty-seven thousand, three hundred forty-four ______

On your own paper, write the following in words.

⑬ 5,003,713

⑭ 106,070,000,005

⑮ 100,465,200

⑯ 63,000,000,010

Maintaining Skills

(Answers given are not in correct order.)

① $14312 + 5032 + 10 + 689 =$

② $70031 - 5894 =$

③ $9042 \times 708 =$

④ $64029 \div 16 =$

⑤ $701.3 + 63.963 + 345 + .47 =$

⑥ $16 - 4.086 =$

⑦ $3.039 \times .004 =$

⑧ $7.21236 \div .012 =$

⑨ $4\frac{7}{8} + 2\frac{2}{5} =$

⑩ $16\frac{2}{7} - 9\frac{3}{4} =$

⑪ $9\frac{1}{3} \cdot 3\frac{4}{5} =$

⑫ $2\frac{5}{6} \div 16\frac{1}{2} =$

⑬ $18 \div \frac{2}{3} =$

⑭ $\frac{3}{7} \cdot \frac{1}{4} \cdot \frac{21}{9} \cdot \frac{16}{18} =$

Answers

ⓐ 27

ⓑ $6\frac{15}{28}$

ⓒ .012156

ⓓ $35\frac{7}{15}$

ⓔ 6401736

ⓕ 20043

ⓖ 601.03

ⓗ $\frac{2}{9}$

ⓘ 1110.733

ⓙ 64137

ⓚ 11.914

ⓛ $\frac{17}{99}$

ⓜ 4001 r 13

ⓝ $7\frac{11}{40}$

Mathematics Objective

3. Use the properties of whole numbers to name a sum, difference, product, or quotient.

Teaching Techniques and Introductory Activities

Properties
For teaching suggestions see page 15, 35, 39 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Adding and subtracting whole numbers
p. 7 (7-11, 18-26)

Multiplying whole numbers
p. 31 (21-31) p. 32 (1-5, 20-25)

Dividing whole numbers
p. 47 (19-24) p. 48 (1-11)

Identity property could be introduced here
a + 0 = 0 + a = a a x 1 = 1 x a = a

Properties
For teaching suggestions see page 44-45 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Addition and subtraction
p. 18 p. 30-37

Multiplication
p. 49 (23-27) p. 50

Division
p. 63 (21-24) p. 64 (40-44) challenge
p. 67-69

Supplementary Activities and Resources

Properties
For teaching suggestions see page 9, 16, 17 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

8 (Scott Foresman)
multiplication p. 10-11
division p. 18-19

8 Silver Burdett
addition p. 7 challlenge p. 13 (16-21)
subtraction p. 15 (26-30)

In Guide: Work Sheets
1. Properties
2. Using the properties of operations
3. Whole numbers
4. cryptograms

MRC
MS B23, B24, B39, B40
MS A26, 27
MAH 4.09, 5.05, 9.03

Properties

Identify the property used:

① $4(1) = 4$

② $3 + (5+4) = (3+5) + 4$

③ $\frac{1}{2} + 0 = \frac{1}{2}$

④ $\frac{1}{3} + \frac{3}{4} = \frac{3}{4} + \frac{1}{3}$

⑤ $.5(7+2) = .5(7) + .5(2)$

⑥ $\frac{3}{7}(13-5) = \frac{3}{7}(13) - \frac{3}{7}(5)$

⑦ $5(7 \cdot 9) = (5 \cdot 7)(9)$

Complete the following properties

⑧ $\left(\frac{1}{4} + \frac{1}{4}\right) + \frac{3}{4} = \frac{1}{4} + \left(\frac{1}{4} + \frac{3}{4}\right)$

$\underline{\qquad} + \frac{3}{4} = \frac{1}{4} + \underline{\qquad}$

$\underline{\qquad} = \underline{\qquad}$

Use the properties to group each expression to solve mentally.

⑨ $225 + 62 + 75 + 38$

⑩ $23 + 17 + 50 + 43$

⑪ $125 \cdot 7 \cdot 3 \cdot 4$

⑫ $27 \cdot 25 \cdot 2 \cdot 8$

Using the Properties of Operations

Example: $4[6+(2+5)] = 4[(2+5)+6]$ commutative

$= [4(2+5)] + 4(6)$ Distributive

$= [4(2)+4(5)] + 4(6)$ Distributive

$= [2(4)+5(4)] + 6(4)$ Commutative

$= 2(4)+5(4)+6(4)$ Associative

Write the name of the property used to obtain the expression from the one preceding it.

① $5[(3+4)+6] =$
$5[3+(4+6)] =$
$5[(4+6)+3] =$
$5(4+6) + 5(3) =$
$5(4) + 5(6) + 5(3) =$

② $6[(8)(5)] =$
$[6(5)]\cdot 8 =$
$[5(6)]\cdot 8 =$
$5(6\cdot 8) =$
$(6\cdot 8)\cdot 5 =$

③ $(4\cdot 7) + (3\cdot 4)$
$(4\cdot 7) + (4\cdot 3)$
$4(7+3)$
$4(3+7)$
$(3+7)4$

Show how you would use the distributive property to do these multiplications mentally.

④ 13×24

⑤ $(11)(99)$

⑥ $(206)(40)$

⑦ $12\cdot 32$

⑧ $(13)(301)$

Whole Numbers

① $66 \cdot 8 =$

② $684 - 308 =$

③ $97 \cdot 3 =$

④ $575 + 37 + 5 + 1006 =$

⑤ $455 \div 13 =$

⑥ $3000 - 1584 =$

⑦ $(235)(35) =$

⑧ $6864 \div 429 =$

⑨ $493 \cdot 7 =$

⑩ $27 + 8 + 106 + 32 =$

⑪ $576 - 21 =$

⑫ $737 \cdot 86 =$

⑬ $189 \div 7 =$

⑭ $21 + 7 + 81 + 206 =$

⑮ $13001 - 476 =$

⑯ $3488 \div 8 =$

⑰ $92 \cdot 157 =$

⑱ $5616 \div 24 =$

⑲ $2967 \div 69 =$

⑳ $15 \cdot 56 =$

㉑ $76 + 817 - 454 =$

Cryptograms

Convert the letters into numerals so that a correct problem in addition is revealed. The Key differs for each problem.

$$\begin{array}{r} \text{S E N D} \\ \text{M O R E} \\ \hline \text{M O N E Y} \end{array} \qquad \begin{array}{r} \text{C R O S S} \\ \text{R O A D S} \\ \hline \text{D A N G E R} \end{array} \qquad \begin{array}{r} \text{D O N A L D} \\ \text{G E R A L D} \\ \hline \text{R O B E R T} \end{array}$$

$$\begin{array}{r} \text{T H I S} \\ \text{I S} \\ \text{V E R Y} \\ \hline \text{E A S Y} \end{array} \qquad \begin{array}{r} \text{W R O N G} \\ \text{W R O N G} \\ \hline \text{R I G H T} \end{array} \qquad \begin{array}{r} \text{J U N E} \\ \text{J U L Y} \\ \hline \text{A P R I L} \end{array}$$

$$\begin{array}{r} \text{S E N D} \\ \text{M O R E} \\ \text{G O L D} \\ \hline \text{M O N E Y} \end{array}$$

Answers

Whole Numbers

① 528
② 376
③ 291
④ 1623
⑤ 35
⑥ 1416
⑦ 8225
⑧ 16
⑨ 3451
⑩ 173
⑪ 545
⑫ 63,382
⑬ 27
⑭ 315
⑮ 12,525
⑯ 436
⑰ 14,444
⑱ 234
⑲ 43
⑳ 840
㉑ 439

Using properties

① Associative
commutative
Distributive
Distributive

② Associative and
commutative
commutative
Associative
Commutative

③ Commutative
Distributive
Commutative
Commutative

④ 13(20 + 4)
⑤ 11(90 + 9)
⑥ 40(200 + 6)
⑦ 12(30 + 2)
⑧ 13(300 + 1)

Properties

① Identity (Mult.)
② Associative (Add.)
③ Identity (Add.)
④ Commutative (Add.)
⑤ Distributive (Mult.
⑥ Distributive over Add. + Sub.)
⑦ Associative (Mult.)
⑧ $\frac{\frac{2}{4}}{\frac{5}{4}}$, $\frac{1}{\frac{5}{4}}$

⑨ (225 + 75) + (62 + 38)
⑩ (23 + 17) + (50 + 43)
⑪ (125 · 4)(7 · 3)
⑫ (25 · 8)(27 · 2)

Cryptograms

$$\begin{array}{r} 9567\\ \underline{1085}\\ 10652 \end{array} \qquad \begin{array}{r} 96233\\ \underline{62513}\\ 158746 \end{array} \qquad \begin{array}{r} 526485\\ \underline{197485}\\ 723970 \end{array}$$

$$\begin{array}{r} 3915\\ 15\\ \underline{4826}\\ 8756 \end{array} \qquad \begin{array}{r} 25938\\ \underline{25938}\\ 51876 \end{array} \qquad \begin{array}{r} 8432\\ \underline{8475}\\ 16907 \end{array}$$

(one of two possibilities)

$$\begin{array}{r} 6054\\ 1720\\ \underline{9734}\\ 17508 \end{array}$$

(one of 32 possibilities)

Mathematics Objective

4. Write a standard numeral for a power of a number
5. Evaluate expressions involving whole number exponents
6. Write a whole number in expanded notation using exponents

Teaching Techniques and Introductory Activities

Exponents and expanded notation
For teaching suggestions see page 26-27 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Evaluating Expressions
p. 36, 38, 49, 51

For teaching suggestions see page 10-11 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Evaluating Expressions
p. 25, 26, 42, 43, 54

Supplementary Activities and Resources

Exponents and Expanded Notation
For teaching suggestions see page 68-69 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

In Guide: Exponents and Expended Notation:
    Explanation of zero power
    Expanded numerals and exponents (Answers)
    A Majic Square

7 H-3 8 practice p. 27
7 practice pp. 9-10 8 L-16

Evaluating Expressions
p. 15, 21 Scott Foresman 8

Exponents and Expanded Notation
For teaching suggestions see p. 8 and p. 28 (Silver Burdett) 8

Evaluating Expressions
p. 29 (Silver Burdett) 8
p. 39 (28-39)
p. 433 (22-31)

MRC:
MS C-16

Exponents and Expanded Notation

Exponents give a convenient way of writing expressions that involve repeated use of a factor.

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$$

$$10 \times 10 = 10^2$$

$$10 = 10^1$$ The first power of any whole number is that whole number.

$$\frac{10^6}{10^2} = \frac{1000000}{100} = 10^4$$

$$\frac{10^5}{10^2} = \frac{100000}{100} = 10^3$$

$$\frac{10^4}{10^2} = \frac{10000}{100} = 10^2$$

$$\frac{10^3}{10^2} = \frac{1000}{100} = 10^1$$

$$\frac{10^2}{10^2} = \frac{100}{100} = 10^0$$ The zero power of any whole number, except 0, is one.

Examine the following patterns to prove that it makes sense to define the zero power of any number to be one.

$$\begin{array}{ccccccc} 2^3 & & 2^2 & & 2^1 & & 2^0 \\ 8 & \xrightarrow{\div 2} & 4 & \xrightarrow{\div 2} & 2 & \xrightarrow{\div 2} & 1 \end{array}$$

$$\begin{array}{ccccccc} 3^3 & & 3^2 & & 3^1 & & 3^0 \\ 27 & \xrightarrow{\div 3} & 9 & \xrightarrow{\div 3} & 3 & \xrightarrow{\div 3} & 1 \end{array}$$

$10^0 = 1$ should be used when expanding numerals using exponents.

$$5{,}639{,}712 = 5 \cdot 10^6 + 6 \cdot 10^5 + 3 \cdot 10^4 + 9 \cdot 10^3 + 7 \cdot 10^2 + 1 \cdot 10^1 + 2 \cdot 10^0$$

$$7{,}000{,}004 = 7 \cdot 10^6 + 4 \cdot 10^0$$

Expanded Numerals and Exponents

Write the standard numeral for each of the following:

① $(4 \cdot 10^6) + (3 \cdot 10^5) + (2 \cdot 10^4) + (1 \cdot 10^3) + (7 \cdot 10^2) + (8 \cdot 10^1) + (1 \cdot 10^0)$

② $7 \cdot 10^5 + 6 \cdot 10^4 + 5 \cdot 10^1 + 9 \cdot 10^0$

③ $8 \cdot 10^6 + 5 \cdot 10^5 + 7 \cdot 10^2 + 3 \cdot 10^1$

④ $3 \cdot 10^7 + 4 \cdot 10^3$

Write each of the following in expanded form using exponents:

① 3895

② 156,901

③ 70,600,000

④ 6,283,719

⑤ 8,000,004

Simplify each of the following

① $4^3 - 3^3 =$
② $3 \times 10^3 =$
③ $10^6 \div 10^3 =$
④ $2^2 \times 5^2 =$
⑤ $1^5 \times 2^3 =$
⑥ $1^7 + 1^8 + 1^3 =$
⑦ $2^4 \cdot 5^2 =$
⑧ $2^6 =$
⑨ $(17^2)(1^4) =$
⑩ $7^2 + 8^2 =$
⑪ $4^3 \div 2^3 =$
⑫ $1^{11} \times 1^{12} \times 1^{60} =$
⑬ $(4 \cdot 5)^2 =$
⑭ $4^2 \cdot 5^2 =$
⑮ $3^3 \cdot 5^1 =$
⑯ $\left(\frac{1}{4}\right)^2 =$
⑰ $\left(\frac{3}{8}\right)^2 \cdot \left(\frac{1}{5}\right)^2 =$
⑱ $(.06)^3 + (.7)^2 =$
⑲ $2^5 + (.1)^2$

Which number is greater?

⑳ 2^3 or 3^2
㉑ 4^3 or 3^4
㉒ 1^5 or 5^1
㉓ 2^3 or $2 \cdot 3$
㉔ 5^2 or $5 \cdot 2$
㉕ 3^3 or $3 \cdot 3$

㉖ 5^3 or 3^5
㉗ 6^2 or 3^3
㉘ 4^2 or 2^3
㉙ 10^2 or 5^3

Evaluate the following expressions for the given replacements.

㉚ $3a^2$; $a = 7$

㉛ $(3a)^2$; $a = 7$

㉜ $4a^2b^3$; $a = 2$, $b = 3$

㉝ $(4a)^2b^3$; $a = 2$, $b = 3$

㉞ $4(a \cdot b)^2$; $a = 2$, $b = 3$

㉟ $4a^2b^2$; $a = 2$, $b = 3$

㊱ $(4a \cdot b)^2$; $a = 2$, $b = 3$

㊲ $a^2 + b^3$; $a = 2$, $b = 3$

Write with exponents

㊳ $5a \cdot 3a =$

㊴ $6b \cdot 4a =$

㊵ $aaa(aa) =$

Answers
Expanded Numerals and Exponents

① 4,321,781
② 760,059
③ 8,500,730
④ 30,004,000

① $3\cdot10^3 + 8\cdot10^2 + 9\cdot10^1 + 5\cdot10^0$
③ $7\cdot10^7 + 6\cdot10^5$
② $1\cdot10^5 + 5\cdot10^4 + 6\cdot10^3 + 9\cdot10^2 + 1\cdot10^0$
④ $6\cdot10^6 + 2\cdot10^5 + 8\cdot10^4 + 3\cdot10^3 + 7\cdot10^2 + 1\cdot10^1 + 9\cdot10^0$
⑤ $8\cdot10^6 + 4\cdot10^0$

① 37
② 3000
③ 10^3
④ 100
⑤ 8
⑥ 1
⑦ 400
⑧ 64
⑨ 289
⑩ 113

⑪ 8
⑫ 1
⑬ 400
⑭ 400
⑮ 135
⑯ $\frac{1}{16}$
⑰ $\frac{9}{1600}$
⑱ .490216
⑲ 32.01

p. 2
⑳ 3^2
㉑ 3^4
㉒ 5^1
㉓ 2^3
㉔ 5^2
㉕ 3^3
㉖ 3^5
㉗ 6^2
㉘ 4^2
㉙ 5^3

㉚ 147
㉛ 441
㉜ 432
㉝ 1728
㉞ 144
㉟ 144
㊱ 576
㊲ 31
㊳ $15a^2$
㊴ $24ab$
㊵ a^5

A Magic Square

$2^5 + 4^2$	$2^2 + 2^1$	3^2	$8^2 - 5^2$
$4^2 - 1^{17}$	$7^2 - 2^4$	$5^2 + 5^1$	$5^2 - 9^0$
3^3	$5^2 - 2^2$	$3^3 - 3^2$	$3^3 + 3^2$
$2^3 + 2^2$	$7^2 - 7$	$7^2 - 2^2$	$15^2 - 12^2 = n^2$ $n =$

What is the sum of each column, row, and diagonal?

____

A Magic Square

48	6	9	39
15	33	30	24
27	21	18	36
12	42	45	3

Sum 102

A 3-by-3 magic square can be constructed using the following patterns.

a - x	a+x-y	a+y
a+x+y	a	a-x-y
a-y	a-x+y	a+x

Note: The sum of x+y should always be less than a when making a magic square with whole numbers.

Mathematics Objective

7. Use the correct order of operations to evaluate an expression

Teaching Techniques and Introductory Activities

Order of operations
For teaching suggestions see page 50 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

For teaching suggestions see page 51 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

For teaching suggestions see page 55 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Supplementary Activities and Resources

Order of operation
For teaching suggestions see page 20 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Evaluating expressions
For teaching suggestions see page 21 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

7 Practice p. 25	7 HA p. 6
8 Practice p. 12	8 HA p. 4

For teaching suggestions see page 230 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

In Guide: Order of Operations worksheet
Answers
Activity: The Day of The Week

MT - Calculator Crossword Puzzle
Activities - pp. 279-282, April, 1978

ORDER OF OPERATIONS

A symbol of inclusion is used to enclose, or include, an expression for a particular number. Parenthes, brackets, braces and the bar are used for this purpose.

Parentheses	Brackets	Braces	Bar
$3(4+2)$	$3[4+2]$	$3\{4+2\}$	$\overline{4+2}$

In working with fractions you have seen that the bar acts as a division sign, as well as a symbol of inclusion. For example, in the expression below, the bar groups the 18 and the 6; it also groups the 4 and the 1. The bar tells you that the number (18 - 6) is to be divided by the number (4 - 1)

$$\frac{18-6}{4-1} = \frac{12}{3} = 4$$

For a grouping inside another grouping, such as $4[3+(6\times5)]$, always simplify the numeral in the innermost symbol of inclusion and proceed to work toward the outermost grouping until all symbols of inclusion are removed.

$$\begin{aligned} 4[3+(6\times5)] &= 4[3+30] \\ &= 4[33] \\ &= 132 \end{aligned}$$

In the absence of grouping symbols the order of operations is as indicated from left to right:

1. first, find powers and roots;
2. next, multiply and divide;
3. last, add and subtract

Order of Operations

Write parentheses or brackets, or both, in each of the following so that it becomes a name for the number after it.

① $9+6\times 8$ 120
② $9+6\times 8$ 57
③ $36-9\div 3+6$ 15
④ $36-9\div 3+6$ 3
⑤ $36-9\div 3+6$ 39
⑥ $36-9\div 3+6$ 35
⑦ $2\frac{1}{2}-\frac{3}{4}\times\frac{1}{2}+\frac{1}{4}$ $\frac{15}{8}$
⑧ $2\frac{1}{2}-\frac{3}{4}\times\frac{1}{2}+\frac{1}{4}$ $\frac{19}{8}$
⑨ $2\frac{1}{2}-\frac{3}{4}\times\frac{1}{2}+\frac{1}{4}$ $\frac{31}{16}$
⑩ $2\frac{1}{2}-\frac{3}{4}\times\frac{1}{2}+\frac{1}{4}$ $\frac{9}{8}$
⑪ $22\div 4-2+19$ 30

Use order of operations to solve the following:

⑫ $6\cdot 7+12\div 3+2=$
⑬ $[32-(48-8\cdot 5)]\div 6=$
⑭ $12\div 3\times 4\div 8=$
⑮ $18-6\div 2\times 3=$
⑯ $5\cdot 6+8\cdot 4\div 16=$
⑰ $7+9(4-3)=$
⑱ $23+18(11+24)=$
⑲ $1\times 7\div 7+1-8\div 8=$
⑳ $28-14\div 7+2\times 10+40=$

㉑ $\dfrac{3[8-(4+2)]+2}{36-[14+6(3)]}=$

㉒ $\left[\dfrac{18-(8\div 4)}{(3+1)(7-5)}\right]\cdot[3]=$

㉓ $\dfrac{13(4)}{9+4}+\dfrac{12(3)-4}{4(7-5)}=$

㉔ $\dfrac{6(2)+[5(3)-2(3)]}{[13-(12-6)]}=$

㉕ $\left[\dfrac{4(8)-6(2)}{(3+2)(8-4)}\right][5]=$

㉖ $\left[\dfrac{8(3)}{4(11-5)}\right]\cdot\left[\dfrac{(11-5)2}{4}\right]=$

㉗ $\dfrac{8(3)+[4(3)-5(2)]}{2[17-(12-8)]}=$

Answers

Order of Operations

① $(9+6)\times 8$
② $9+(6\times 8)$
③ $(36-9)\div 3+6$
④ $36-9\div(3+6)$
⑤ $36-(9\div 3)+6$
⑥ $36-[9\div(3+6)]$
⑦ $2\frac{1}{2}-[(\frac{3}{4}\times\frac{1}{2})+\frac{1}{4}]$
⑧ $[2\frac{1}{2}-(\frac{3}{4}\times\frac{1}{2})]+\frac{1}{4}$
⑨ $2\frac{1}{2}-[\frac{3}{4}\times(\frac{1}{2}+\frac{1}{4})]$
⑩ $[(2\frac{1}{2}-\frac{3}{4})\times\frac{1}{2}]+\frac{1}{4}$
⑪ $[22\div(4-2)]+19$

⑫ 48
⑬ 4
⑭ 2
⑮ 9
⑯ $31\frac{1}{2}$
⑰ 16
⑱ 653
⑲ 1
⑳ 86
㉑ 2
㉒ 6
㉓ 8
㉔ 3
㉕ 5
㉖ 3
㉗ 1

THE DAY OF THE WEEK

PURPOSE

To introduce an attention getting algorithm in use many years ago. This algorithm can be used to calculate the day of the week any date falls on for the years 1 through 3000 A.D.

METHOD

To the last two digits of the year of the date in question, add ½ of those digits. Disregard any fractional parts. To this sum add the day of the month and the index of the month and century. (The latter two numbers can be found in the charts below.) Finally, divide the above result by 7. The remainder is the day of the week, counting Sunday the 1st, Monday the 2nd, etc.

INDEX OF MONTHS

June	0
Sept. & Dec.	1
April & July	2
Jan. & Oct.	3
May	4
August	5
March, Feb. & Nov.	6

Note: In leap years the index of Jan. is 2 and the index of Feb. is 5. There are no other changes.

CENTENNIAL INDEX

	Index
200 900 1800 2200 2600 3000	0
300 1000	6
400 1100 1900 2300 2700	5
500 1200 1600 2000 2400 2800	4
600 1300	3
000 700 1400 1700 2100 2500 2900	2
100 800 1500	1

Examples:

I. The United States declared her independence from England on July 4, 1776. What day of the week was July 4, 1776?

Solution:
76 (last 2 digits of given year)
19 (½ of last 2 digits)
4 (day of month)
2 (index for July)
2 (index for century)
103 Sum

103 ÷ 7 gives remainder of 5.

Hence, July 4, 1776, was a Thursday.

II. President Lincoln was assassinated on April 14, 1865. What day of the week was he killed?

Solution:
65 (last 2 digits of given year)
16 (¼ of last 2 digits--note remainder is dropped)
14 (day of the month)
2 (index of April)
0 (index of century)
97 Sum

97 ÷ 7 gives a remainder of 6.

Hence, Lincoln was killed on a Friday.

One final comment: If the remainder upon division by 7 is 0, the day of the week is Saturday.

REFERENCES

Practical Arithmetic, R. J. Woodward, Becktold Printing and Book Manufacturing Co., St. Louis, Missouri, 1894.
Puzzles and Paradoxes, T. H. O'Beirne, Oxford University Press.

activities

Edited by **Evan M. Maletsky,** *Montclair State College, Upper Montclair, New Jersey*
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CALCULATOR CROSSWORD PUZZLE

By Joseph F. Goodhue, Newport High School, Newport, NH 03773

Teacher's Guide

Grade Level: 7–12.

Materials: Eight-place pocket calculator (preferably with a memory).

Objectives: In this activity students use pocket calculators to demonstrate their knowledge of the correct order of operation for a given mathematical expression.

Directions: The first worksheet provides some warmup exercises that review the rules for the order of operations and establishes the procedure for spelling out words by inverting the calculator. In general, multiplication and division take priority over addition and subtraction unless symbols of grouping (parentheses, brackets, and braces) are involved. The rule for the order of operations is as follows:

> *First carry out all operations within symbols of grouping. Then perform multiplications and divisions in the order in which they occur from left to right. Finally, perform all additions and subtractions.*

For each entry in the crossword puzzle (sheet 2), the student performs the indicated computation, gets a result, and then inverts the calculator to read a word or abbreviation. The word or abbreviation is checked against the verbal cue and then entered into the appropriate cells in the crossword puzzle diagram.

Looking at 1 Across on the crossword puzzle (sheet 2), we need a four-letter word for "legendary Viking." Entering the expression $5(200 \times 15 - 10 \times 172 - 6)$ on the calculator in the correct order of operation should produce the number

6 3 7 0.

Inverting the calculator, we read the name

0 L E 9.

which we write in the given spaces in our puzzle.

The answer sheet (sheet 3) can be reproduced for the students if desired.

REFERENCE

Vannatta, Glen D., A. Wilson Goodwin, and Harold P. Fawcett. *Algebra Two: A Modern Course.* Columbus, Ohio: Charles E. Merrill, 1966.

This section is designed to provide mathematical activities suitable for reproduction in worksheet and transparency form for classroom use. Appropriate laboratory experiences, discovery activities, and model constructions of mathematical significance drawn from the topics of the seventh, eighth, and nine grades are most welcome. Send your suggestions, in a four-page format similar to the "Activities" already published, to the managing editor for review.

ORDER OF OPERATIONS

Sheet 1

1. In each of the following, use a calculator and the rule for the order of operations to compute the given expression. When you get an answer, turn your calculator upside down and read the word. Record intermediate steps and the word.

Example: 123(45 + 72) - 40584 ÷ 6 + 13 x 7
= 123 x 117 - 40584 ÷ 6 + 13 x 7
= 14391 - 6764 + 91
= 7718
B I L L

a) (10+13(236+587)-595x5)÷10 000

b) 37+(20+(19+13x41-5))

c) 96(78-24÷8-25)+24x25-91

d) 623x411-213x303+1296÷4x579-288-16

2. Write the letters that correspond to the numerals:

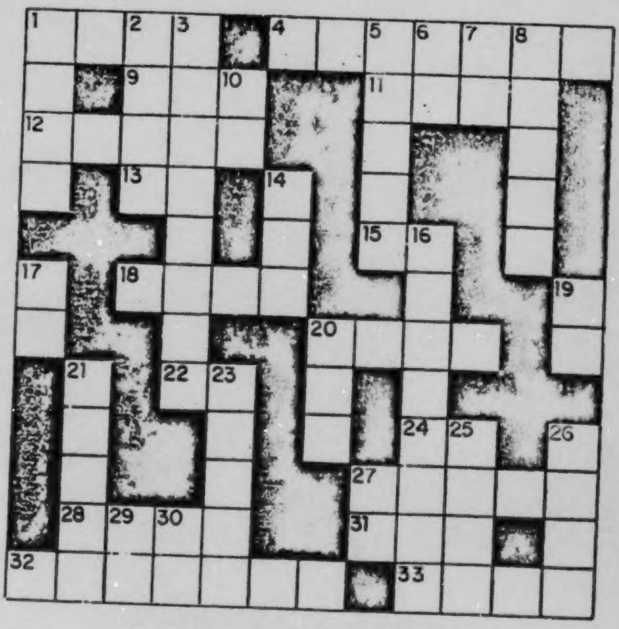
0	1	2	3	4	5	6	7	8	9

3. Make your calculator read these words:

	Keys to Press
SIZE	
gIggLE	
hOBBLE	
BOBSLED	

CALCULATOR CROSSWORD PUZZLE

Sheet 2



ACROSS

1. Legendary Viking
   5(200x15-10x172-6)
4. East African river
   5(115889x3+2)
9. Actor Hope
   (12x199+36)÷3
11. African desert
   (350x20+40x5)÷4+6
12. Rumanian river
   15(4x895+3x3)
13. Siberian river
   (14x20x5+40)÷(5x3+3)
15. Actor Greene (initials)
   2x15-5x2+5x8+7
18. ______Horizonte
   0.8856÷12
20. Swiss lake
   2(9x7x58+5)
22. Filmmaker Goldwyn (initials)
   (38x14+11x4)÷8-8+1
24. Actor Bridges
   (16.6-6.2)÷(5x30-20)
27. Belgian river
   25x20(35x2+1)+37
28. Dutch painter
   8x25(4x5+3)+6
31. State
   (50(7x4+5))÷3+221
32. Nigerian Mountain Range
   15(300x320+1223)
33. Make dirty
   5x27(9x7-10)-50

DOWN

1. French river
   10((70x10)÷2+1)
2. _____of Reims
   4x0.0220+0.0003
3. Nazi minister (WW II)
   88888888-31500888+150x2+6
5. German sculptor
   (313254+305505+5x29)+8
6. Actor Bridges
   0.80÷16+0.03
7. Hitler's mistress (initials)
   8x7+9x3
8. Belgian University
   36007+15.5x20
10. College Degree
   (350+32x0.3)÷6.2
14. Self 0.08x0.9-0.009
16. Same as 3 Down
17. Old Saxon (abbr.)
   (9x39-1)÷7
19. Old Latin (abbr.)
   3(8x21-4)÷6+1-13
20. Little ____ Horn
   22x32-16x6+10
21. Capt. of H.M.S. *Bounty*
   2((14x12+3)x(15x9)+4
23. Belonging to 28 Across
   6(2733÷3x10-9)÷10000
25. Norwegian city
   0.1230-0.0340-0.0140
26. Telephone inventor
   15+8x8x11x11-21
27. Long Island (abbr.)
   (21x14-5)÷17
29. Old English (abbr.)
   (645÷15+47)÷3
30. Mathematician Boole (initials)
   (12x79-2)÷(30÷6+6)

ANSWER SHEET

Sheet 1:

1. a) 0.7734 BILL b) 604 hOg
   c) 5309 GOES d) 378806 gOBBLE

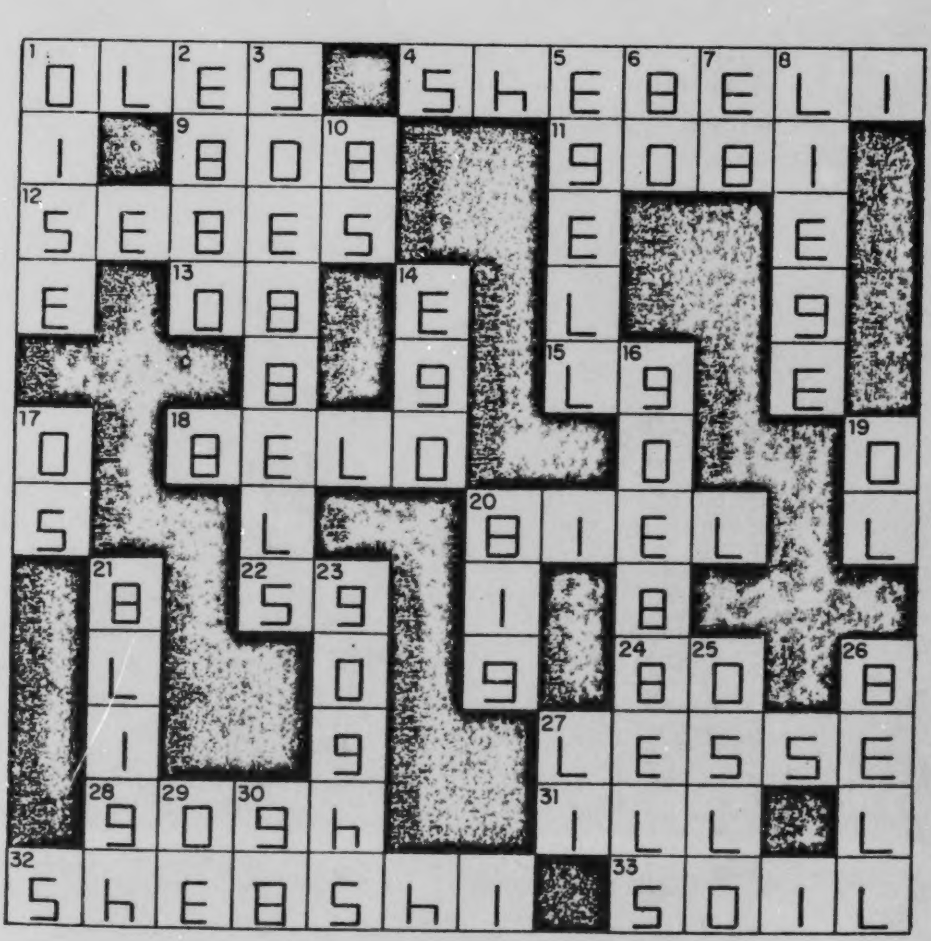
2.

0	1	2	3	4	5	6	7	8	9
O or D	I	Z	E	h	S	g	L	B	G

3.

SIZE	3215
gIgglE	376616
hOBBLE	378804
BOBSLED	.375808

Sheet 2:



1 O	L	2 E	3 G		4 S	h	5 E	6 B	7 E	8 L	I
I		9 B	O	10 B			11 G	O	B	I	
12 S	E	B	E	S			E			E	
E		13 O	B		14 E		L			G	
			B		G		15 L	16 G		E	
17 O		18 B	E	L	O			O			19 O
S			L			20 B	I	E	L		L
	21 B		22 S	23 G		I		B			
	L			O		G		24 B	25 O		26 B
	I			G			27 L	E	S	S	E
	28 G	29 O	30 G	h			31 I	L	L		L
32 S	h	E	B	S	h	I		33 S	O	I	L

Mathematics Objective

8. write an expression which represents a verbal statement involving the word or word phrse: sum difference more than, less than, increase, decrease
9. solve addition and subtraction equations

Teaching Techniques and Introductory Activities

Translating expressions
For teaccing suggestions see page 16-17 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Solving addition and Subtraction equations
For teaching suggestions see page 20-23 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Writing expressions
For teaching suggestions see page 25 of the Teacher's Edition of *Mathematics Around Us* 7 (Silver Burdett)

Solving addition and subtraction equations
For teaching suggestions see page 27 of the Teacher's Edition of *Mathematics Around Us* 7 (Silver Burdett)

Supplementary Activities and Resources

Expressions
For teaching suggestions see page 26-27 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Solving equations
For teaching suggestions see page 28-31 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

7 practice p. 6, 7	7H-2
8 practice p. 13, 14, 15	8L-9, 8H-5

Writing
For teaching suggestions see page 16 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

Solving
For teaching suggestions see page 17-20 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

In Guide: Worksheet - expressions
Inverse Operations
Inverse Operation of Addition (Key)
Maintaining Skills

MRC:
MS B 30

Mathematics Objective

10. Write an expression which represents a verbal statement involving the word or word phrase: product, quotient, times, divided by
11. Solve equations using multiplication and division

Teaching Techniques and Introductory Activities

Writing expressions
For teaching suggestions see page 52-53 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Solving equations
For teaching suggestions see page 54-55 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)

Inverse operation
For teaching suggestions see pag 56 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)

No material in 7SB on the above

Supplementary Activities and Resources

Writing expressions
For teaching suggestions see page 34-35 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Solving equations
For teaching suggestions see page 36-43 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)

7 practice p. 26, 27 8 LA p. 10, p. 11
8 practice p. 16, 17, 18 HA p. 6

Solve equations
For teaching suggestions see page 36 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)

Word problems p. 37-41
Skillmaster 8-8, 8-9, 8-10

MRC
AWA IV 24, 25

Write an expression for each word phrase.

① The sum of 5 and 8
② The difference when x is subtracted from 10.
③ A number decreased by 4.
④ The quotient when a number is divided by 6
⑤ 5 less than the sum of a number and 7
⑥ The product of 7 and the sum of a number and 3
⑦ 6 more than twice a number
⑧ The product of a number and 9 increased by 5
⑨ The sum of a number and 8 less 3
⑩ The sum of 6 and the product of 4 and a number
⑪ The number of cents in x nickels
⑫ One-half of a number decreased by 4
⑬ The square of the sum of x and 4
⑭ The average of a number and 7
⑮ The quotient when the sum of 5 and a number is divided by 7
⑯ 5 times the difference when 3 is subtracted from a number
⑰ The number of cents in d dimes
⑱ 7 less than three times a number

Inverse Operations

Subtraction and addition are inverse operations. Addition is the renaming of two addends as a sum; subtraction is the renaming of a sum and an addend as the other addend. The missing addend is also called a difference. Adding and subtracting the same number "undo" one another. Their combined effect is the same as adding zero.

addend + addend = sum

$7 + 4 = 11$

$$6 + \overbrace{3 - 3}^{0} = 6$$

Every number sentence showing addition has two related subtraction sentences.

addend + addend = sum

$5 + 4 = 9$

S − A = A

$9 - 5 = 4$

S − A = A

$9 - 4 = 5$

Division and multiplication are inverse operations. Multiplication is the renaming of two factors as a product; division is the renaming of a product and a factor as the other factor. The missing factor is also called a quotient. Division "undoes" multiplication.

factor factor product

$6 \times 7 = 42$

$$6 \times \overbrace{7 \div 7}^{1} = 6$$

Every number sentence showing multiplication has two related division sentences.

factor × factor = product

$9 \times 2 = 18$

P ÷ F = F

$18 \div 9 = 2$

P ÷ F = F

$18 \div 2 = 9$

Addition - subtraction:

$n + 4 = 9$

$9 - n = 4$

$9 - 4 = n$

Multiplication - division

$8 \cdot n = 32$

$32 \div n = 8$

$32 \div 8 = n$

$8 \cdot n = 32$

$\frac{32}{n} = 8$

$\frac{32}{8} = n$

Inverse Operation of Addition

Write a subtraction sentence for each addition sentence. Write an addition sentence for each subtraction sentence.

① $x + 16 = 37$ ________

② $59 - 17 = 42$ ________

③ $54 + 78 = 132$ ________

④ $x + 9 = z$ ________

⑤ $52 - m = 34$ ________

⑥ $a + b = c$ ________

⑦ $28 + 60 = 88$ ________

⑧ $n - 47 = 16$ ________

⑨ $a - b = c$ ________

⑩ $(a + 7) - b = 9$ ________
[Keep (a+7) as a quantity]

⑪ $(n - 4) + 10 = n$ ________
[same as above]

Inverse Operation of Multiplication

Write a multiplication problem for each division problem. Write a division problem for each multiplication problem.

① $x \div y = 7$ ________

② $72 = 12 \cdot 6$ ________

③ $7n = 28$ ________

④ $n \cdot m = 36$ ________

⑤ $\frac{144}{18} = 8$ ________

⑥ $3(n) = 15$ ________

⑦ $\frac{a}{b} = c$ ________

⑧ $\frac{n}{6} = 18$ ________

⑨ $(18)(12) = 216$ ________

⑩ $\frac{210}{n} = 14$ ________

⑪ $6(a + b) = 12$ ________

⑫ $\frac{(a + b)}{c} = d$ ________

Inverse Operation of Addition

(Key)

① $x = 37 - 16$ or $16 = 37 - x$

② $59 = 42 + 17$

③ $54 = 132 - 78$ or $78 = 132 - 54$

④ $x = z - 9$ or $9 = z - x$

⑤ $52 = 34 + m$

⑥ $a = c - b$ or $b = c - a$

⑦ $28 = 88 - 60$ or $60 = 88 - 28$

⑧ $n = 16 + 47$

⑨ $a = c + b$

⑩ $(a + 7) = 9 + b$

⑪ $(n - 4) = m - 10$ or

$10 = m - (n - 4)$

Inverse Operation of Multiplication

① $x = 7y$

② $72 \div 12 = 6$ or $\frac{72}{12} = 6$ or $\frac{72}{6} = 12$

③ $n = \frac{28}{7}$ or $7 = \frac{28}{n}$

④ $n = \frac{36}{m}$ or $m = \frac{36}{n}$

⑤ $144 = 8 \cdot 18$

⑥ $n = \frac{15}{3}$ or $3 = \frac{15}{n}$

⑦ $a = b \cdot c$

⑧ $n = 6 \cdot 18$

⑨ $18 = \frac{216}{12}$ or $12 = \frac{216}{18}$

⑩ $210 = 14n$

⑪ $6 = \frac{12}{(a+b)}$ or $(a+b) = \frac{12}{6}$

⑫ $(a + b) = c \cdot d$

Maintaining Skills

(Answers given are not in correct order)

1) 63421 + 63 + 9745 + 8 =
2) 90041 − 6987 =
3) 6000 − 4098 =
4) 3076 × 5004 =
5) 53,721 ÷ 132 =
6) 86,666 ÷ 921 =
7) $6\frac{2}{9} + 1\frac{3}{4} =$
8) $16 - 9\frac{5}{6} =$
9) $7\frac{1}{3} - 3\frac{5}{7} =$
10) $3\frac{1}{2} \cdot 4\frac{1}{6} =$
11) $3\frac{1}{2} \div 4\frac{1}{6} =$
12) $6\frac{1}{5} \cdot 2\frac{3}{4} =$
13) $6\frac{1}{5} \div 2\frac{3}{4} =$
14) 631.3 + 56.45 + 7 + .0004
15) 67.3 − 4.0763 =
16) 9.456 − .97 =
17) 16.4 × .006 =
18) 6.0036 ÷ 1.2 =
19) .908136 ÷ .0009 =

Answers

ⓐ 406 r. 129
ⓑ $6\frac{1}{6}$
ⓒ 83054
ⓓ 63.2237
ⓔ 1009.04
ⓕ 15,392,304
ⓖ 5.003
ⓗ $17\frac{1}{20}$
ⓘ 73237
ⓙ .0984
ⓚ $3\frac{13}{21}$
ⓛ 8.486
ⓜ 94 r. 92
ⓝ $\frac{21}{25}$
ⓞ $14\frac{7}{12}$
ⓟ 694.7504
ⓠ 1902
ⓡ $2\frac{14}{55}$
ⓢ $7\frac{35}{36}$

Mathematics Objective

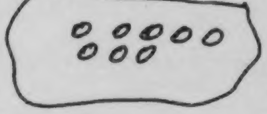
12a. Identify numerals in other bases

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Side trip - Base Two p. 122-123 7 (Scott Foresman)	In Guide: Other numeral systems translating into Base Ten Translating from Base Ten to other bases Additonal Table for Base Twelve Crypotograms 8HA p. 8 To analyze other bases For teaching suggestions see page 71 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) MRC MS C-20, C-21, C-22, C-24, C-25

Other numeral Systems

In our Hindu-Arabic numeral system we group by tens. Ours is a decimal numeral system. We use the digits '0', '1', '2', '3', '4', '5', '6', '7', '8', '9' and combinations of these to record how many. Because we group by tens we use a base ten system.

In a base five system we group by fives. How many is recorded by the symbols '0', '1', '2', '3', '4'. This collection would be recorded as 13_{five} because there is one group of five and three left over.



There is no single digit for ten in our base ten system; instead we use place value along with '1' and '0' to show 1 group of ten and 0 units, or '10'. The base five system has no single symbol for five (such as '5'), but uses '1', '0', and place value in the numeral '10_{five}' to show 1 group of five and 0 units. '10_{five}' is read 'one zero base five'. '13_{five}' is read 'one three base five.'

Compare the following place value charts:

base ten

10,000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$
10^4	10^3	10^2	10^1	10^0	$\frac{1}{10^1}$	$\frac{1}{10^2}$

base five

625	125	25	5	1	$\frac{1}{5}$	$\frac{1}{125}$
5^4	5^3	5^2	5^1	5^0	$\frac{1}{5^1}$	$\frac{1}{5^2}$

base three

81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$
3^4	3^3	3^2	3^1	3^0	$\frac{1}{3^1}$	$\frac{1}{3^2}$

Addition Table for Base Twelve

+	0	1	2	3	4	5	6	7	8	9	t	e	
0	0	1	2	3	4	5	6	7	8	9	t	e	. . .
1	1	2	3	4	5	6	7	8	9	t	e	10	. . .
2	2	3	4	5	6	7	8	9	t	e	10	11	. . .
3	3	4	5	6	7	8	9	t	e	10	11	12	. . .
4	4	5	6	7	8	9	t	e	10	11	12	13	. . .
5	5	6	7	8	9	t	e	10	11	12	13	14	. . .
6	6	7	8	9	t	e	10	11	12	13	14	15	. . .
7	7	8	9	t	e	10	11	12	13	14	15	16	. . .
8	8	9	t	e	10	11	12	13	14	15	16	17	. . .
9	9	t	e	10	11	12	13	14	15	16	17	18	. . .
t	t	e	10	11	12	13	14	15	16	17	18	19	. . .
e	e	10	11	12	13	14	15	16	17	18	19	1t	. . .
	.	.	.	.	.	.	.	.	.	.	.	.	.
	:	:	:	:	:	:	:	:	:	:	:	:	:

Translating into Base Ten

① 421_{five} means $\begin{array}{c|c|c} 25 & 5 & 1 \\ \hline 4 & 2 & 1 \end{array} = 4(25)+2(5)+1(1) = 100+10+1 = 111_{ten}$

② $121_{three} = \begin{array}{c|c|c} 9 & 3 & 1 \\ \hline 1 & 2 & 1 \end{array} = 1(9)+2(3)+1(1) = 9+6+1 = 16_{ten}$

③ $3042_{five} =$

④ $101_{three} =$

⑤ $212_{three} =$

⑥ $333_{five} =$

⑦ $542_{six} =$

⑧ $300_{four} =$

⑨ $1101_{two} =$

⑩ $601_{seven} =$

Translating from Base Ten to Other Bases

① 43_{ten} = 1121 three

43_{ten} =

81	27	9	3	1
0	1	1	2	1

$$\begin{array}{rl} 43 & \\ -27 & 1(27) \\ \hline 16 & \\ -9 & 1(9) \\ \hline 7 & \\ -6 & 2(3) \\ \hline 1 & \end{array}$$

② 17_{ten} = 120 three

17_{ten} =

27	9	3	1
0	1	2	0

$$\begin{array}{rl} 17 & \\ -9 & 1(9) \\ \hline 6 & \\ -6 & 2(3) \\ \hline 0 & \end{array}$$

③ 49_{ten} = 144 five

49_{ten} =

125	25	5	1
0	1	4	4

$$\begin{array}{rl} 49 & \\ -25 & 1(25) \\ \hline 24 & \\ -20 & 4(5) \\ \hline 4 & \end{array}$$

④ 65_{ten} = ________ five

⑤ 59_{ten} = ________ three

⑥ 81_{ten} = ________ five

⑦ 20_{ten} = ________ three

Translating into Base Ten

(Key)

③ $3042_{five} = 3(125) + 0(25) + 4(5) + 2(1) = 397_{ten}$

④ $101_{three} = 1(9) + 0(3) + 1(1) = 10_{ten}$

⑤ $212_{three} = 2(9) + 1(3) + 2(1) = 23_{ten}$

⑥ $333_{five} = 3(25) + 3(5) + 3(1) = 93_{ten}$

⑦ $542_{six} = 5(36) + 4(6) + 2(1) = 206_{ten}$

⑧ $300_{four} = 3(16) + 0(4) + 0(1) = 48_{ten}$

⑨ $1101_{two} = 1(8) + 1(4) + 0(2) + 1(1) = 13_{ten}$

⑩ $601_{seven} = 6(49) + 0(7) + 1(1) = 295_{ten}$

Translating from Base Ten to Other Bases

(Key)

④ $65_{ten} = 230_{five}$

⑤ $59_{ten} = 2012_{three}$

⑥ $81_{ten} = 311_{five}$

⑦ $20_{ten} = 202_{three}$

Other Bases

(Key)

⑥ $1110_{(5)}$

⑦ $653_{(7)}$

⑧ $1142_{(5)}$

⑨ $112_{(5)}$

⑩ $112_{(4)}$

⑪ $211_{(4)}$

⑫ $4121_{(5)}$

⑬ $13_{(10)}$

⑭ $1021_{(4)}$

⑮ $104_{(10)}$

⑯ $1200_{(3)}$

Other Bases

①
$$\begin{array}{r} {}^{1\,1} \\ 123_{(4)} \\ +\ \underline{22_{(4)}} \\ 211 \end{array}$$

②
$$\begin{array}{r} 343_{(7)} \\ \underline{+512_{(7)}} \\ 1155_{(7)} \end{array}$$

③
$$\begin{array}{r} 343_{(5)} \\ \underline{-122_{(5)}} \\ 221_{(5)} \end{array}$$

④
$$\begin{array}{r} 531_{(6)} \\ \underline{-342_{(6)}} \\ 245_{(6)} \end{array}$$

⑤
$$\begin{array}{r} {}^{2}131_{(4)} \\ \underline{\times\ 21_{(4)}} \\ 131 \\ \underline{1012} \\ 10311_{(4)} \end{array}$$

⑥
$$\begin{array}{r} 444_{(5)} \\ \underline{+111\phantom{_{(5)}}} \\ {}_{(5)} \end{array}$$

⑦
$$\begin{array}{r} 252_{(7)} \\ \underline{+401_{(7)}} \end{array}$$

⑧
$$\begin{array}{r} 244_{(5)} \\ \underline{+\ 343_{(5)}} \end{array}$$

⑨
$$\begin{array}{r} 411_{(5)} \\ \underline{-244_{(5)}} \end{array}$$

⑩
$$\begin{array}{r} 300_{(4)} \\ \underline{-122_{(4)}} \end{array}$$

⑪
$$\begin{array}{r} 312_{(4)} \\ \underline{-101_{(4)}} \end{array}$$

⑫
$$\begin{array}{r} 232_{(5)} \\ \underline{\times\ 13_{(5)}} \end{array}$$

Translate the following:

⑬ $111_{(3)} =$ ____________ $_{(10)}$

⑭ $73_{(10)} =$ ____________ (4)

⑮ $252_{(6)} =$ ____________ $_{(10)}$

⑯ $45_{(10)} =$ ____________ (3)

Cryptograms

+/-	○	|	∧	△	□	⇧
○	○	|	∧	△	□	⇧
|	|	∧	△	□	⇧	|○
∧	∧	△	□	⇧	|○	||
△	△	□	⇧	|○	||	|∧
□	□	⇧	|○	||	|∧	|△
⇧	⇧	|○	||	|∧	|△	|□

×/÷	|	∧	△	□	⇧	
|	|	∧	△	□	⇧	
∧	∧	□	|○	|∧	|□	
△	△	|○	|△	∧○	∧△	
□	□	|∧	∧○	∧□	△∧	
⇧	⇧	|□	∧△	△∧	□|	

|
+ |

|
+ ∧

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+ △

|
+ □

∧
+ ∧

∧
+ △

∧
+ □

|∧
+ ∧|

|□
+ ∧△

∧□
+ □⇧

□∧|
+ ∧□∧

∧□△
+ □∧□

|∧□∧
+ ∧○|△

∧□⇧|
+ ⇧∧□□

□∧□
|□∧
∧|⇧
+ □|□

∧□∧□
△|○⇧
⇧○□|
+ |□|∧

1.) □ ⌂ 1 ∧
⊕ ⌂ □ □ ⌂

(2) ⌂ 1 ○ 1
⊕ □ □ 1 ○

③ □ 1 ○ 1 ⌂
⊕ ⌂ □ ∧ ○ ∧

4.) □ □ 1 ∧
⊕ △ ∧ ⌂ □ □

⑤ □ 1
⌂ □
∧ ○
⊕ □ ∧

(6) ⌂ 1
○ ∧
∧ □
⊕ ∧ ⌂

(7.) 1 ∧
⊕ □ □

(8) □ ⌂
⊕ □ □

(9) ⌂ + ⌂ + □ + □ + ⌂ + ∧ +
∧ + □ =

(10) ∧ □
⊖ 1 ∧

(11) □
⊖ ∧

(12) □ 1 ⌂
⊖ △ ○ □

(13) □ 1
⊖ 1 □

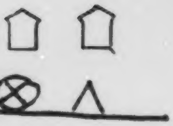
(14). □ ∧
⊖ ∧ ⌂

(15) □ ○ ○ ○
⊖ 1 ∧ ⌂ 1

(16) ⌂ 1 ○ ○
⊖ △ ○ 1 ⌂

(17) □ ○ 1 ○
⊖ ∧ 1 □ ⌂

(18) ⌂ ○ ○ ○
⊖ 1 △ ⌂ □

(19)


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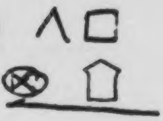

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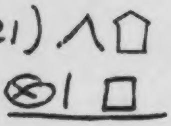

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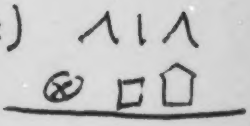

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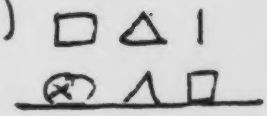

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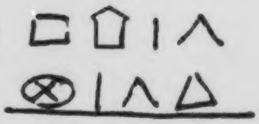

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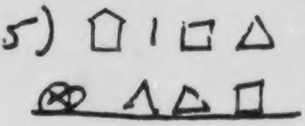

∧ ⟌□　　∧ ⟌　　∧ ⟌⌂

△ ⟌1○　　△ ⟌1△　　△ ⟌1∧

⌂ ⟌∧△⌂　　⌂ ⟌∧⌂△

∧1 ⟌∧□□⌂△△　　□⌂ ⟌□○□△∧○△

Cryptograms

+/-	○	|	∧	△	□	⌂
○	○	|	∧	△	□	⌂
|	|	∧	△	□	⌂	|○
∧	∧	△	□	⌂	|○	||
△	△	□	⌂	|○	||	|∧
□	□	⌂	|○	||	|∧	|△
⌂	⌂	|○	||	|∧	|△	|□

×/÷	|	∧	△	□	⌂	
|	|	∧	△	□	⌂	
∧	∧	□	|○	|∧	|□	
△	△	|○	|△	∧○	∧△	
□	□	|∧	∧○	∧□	△∧	
⌂	⌂	|□	∧△	△∧	□|	

|
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∧□
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□∧|
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||○△

∧□△
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|∧□∧
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△∧⌂⌂

∧□⌂|
+ ⌂∧□□
|∧|△⌂

□∧□
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∧|⌂
+ □|□
|⌂□△

∧□∧□
△|○⌂
⌂○□|
+ |□|∧
∧○□△○

1) □ ⌂ 1 ∧
⊕ ⌂ □ □ ⌂
1 □ □ 0 1

(2) ⌂ 1 0 1
⊕ □ □ 1 0
1 △ ⌂ 1 1

③ □ 1 0 1 ⌂
⊕ ⌂ □ ∧ 0 ∧
1 △ ∧ ∧ 1

4) □ □ 1 ∧
⊕ ∧ ⌂ □ □
1 1 □ 0 0

⑤ □ 1
⌂ □
∧ 0
⊕ □ ∧
∧ □ 1

(6) ⌂ 1
0 ∧
∧ □
⊕ ∧ ⌂
1 ⌂ 0

(7) 1 ∧
⊕ □ □
1 0 0

(8) □ ⌂
⊕ □ □
1 △ △

(9) ⌂ + ⌂ + □ + □ + ⌂ + ∧ +
∧ + □ = ⌂ 1

(10) ∧ □
⊖ 1 ∧
1 ∧

(11) □
⊖ ∧
⌂ ∧

(12) □ 1 ⌂
⊖ △ 0 □
1 1 1

(13) □ 1
⊖ 1 □
∧ △

(14) □ ∧
⊖ ∧ ⌂
1 △

(15) □ 0 0 0
⊖ 1 ∧ ⌂ 1
∧ △ 0 ⌂

(16) ⌂ 1 0 0
⊖ △ 0 1 ⌂
∧ 0 □ 1

(17) □ 0 1 0
⊖ ∧ 1 □ ⌂
1 □ ∧ 1

(18) ⌂ 0 0 0
⊖ 1 △ ⌂ □
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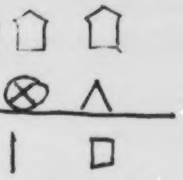

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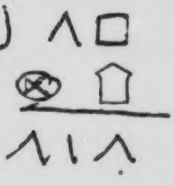

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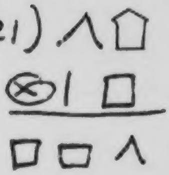

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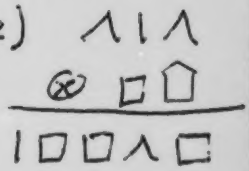

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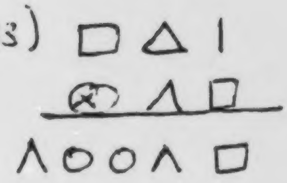

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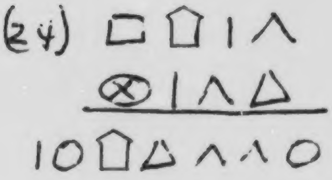

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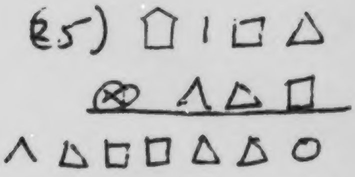

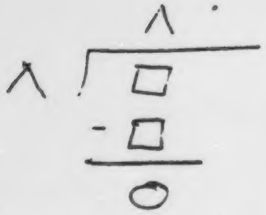

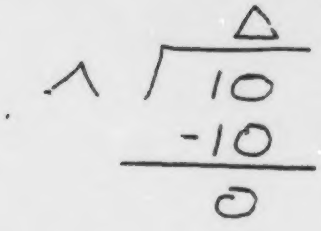

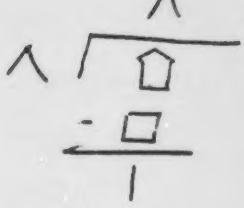

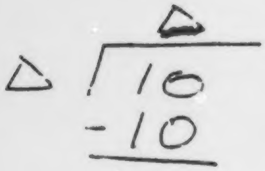

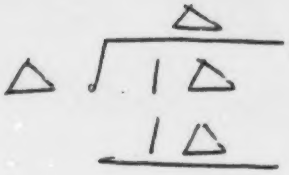

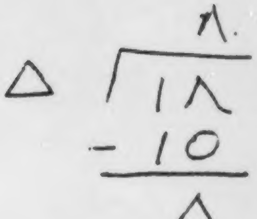

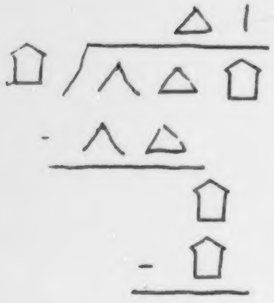

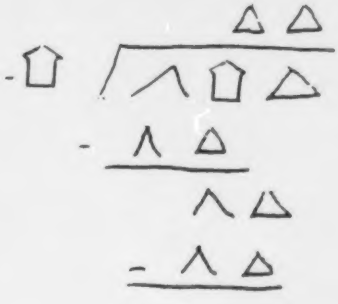

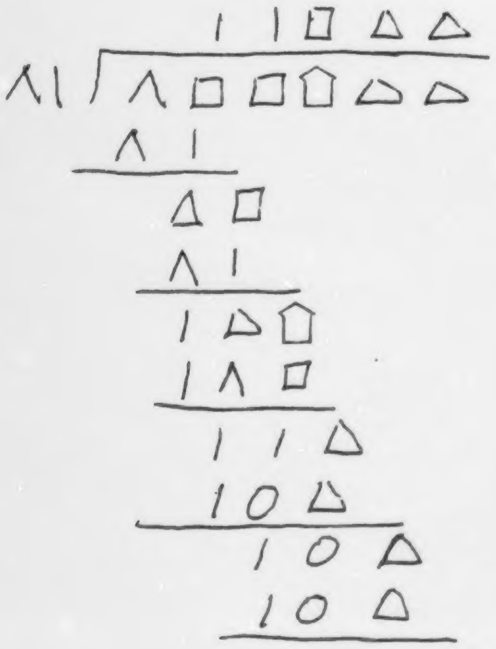

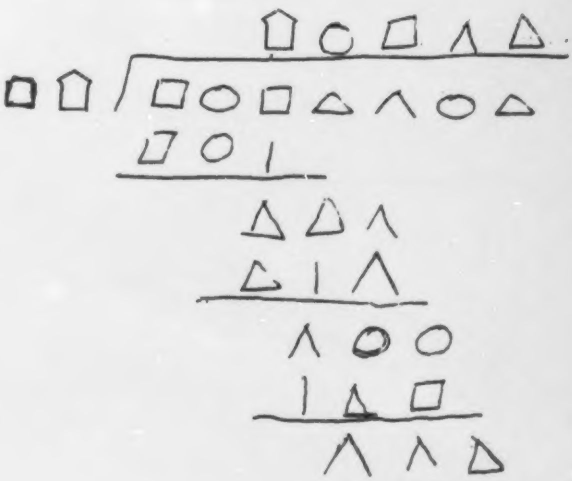

Mathematics Objective

13M Identify prime and composite numbers, identify factors and multiples, and use divisidibilty rules for 2, 3, 4, 5, 6, 9, 10

Teaching Techniques and Introductory Activities

Factors and multiples
For teaching suggestions see page 198-199 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Prime and Composite numbers
For teaching suggestions see page 200-201 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

For teaching suggestions see page 202-203 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Test for divisibility by 4, 8, 6, and 11 are found in teacher's section p. 203.

Factors and multiples
For teaching suggestions see page 72, 74 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Prime and composite numbers
For teaching suggestions see page 76 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Dividisiblilty tests
For teaching suggestions see page 81-82 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Supplementary Activities and Resources

Prime and composite numbers
For teaching suggestions see page 59 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

7 HA p. 20 Fibonacci numbers - (environment)
7 Practice p. 69 (The Sieve of Eroctosthnes
numbers 1-200)
p. 599-602, Oct. '77

MT - Activities - Numbers and Mysticism
pp. 599-602 October, 1977
MT - Activities - Pattern Gazing
pp. 39-40, January, 1979

Factors and multiples
For teaching suggestions see page 62-63 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

Prime & composite
For teaching suggestions see page 65 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

For teaching suggestions see page 68 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

MRC:
MS C32, C33, C43, C36, C37, C38

In Guide: More divisibility rules
Divisibility (worksheet & ans.)

MT - Numbers and Mysticism
Pattern Gazing

More Divisibility Rules

A number is divisible by 4 if the number formed by the last two digits is divisible by 4.

A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

A number is divisible by 6 if the unit digit is even and the sum of the digits is divisible by 3.

To test for divisibility by 11, number the places from the right. Find the sum of the digits in the odd-numbered place. Then find the sum of the digits in the even numbered places. Subtract the smaller number from the larger. If the difference is 0 or divisible by 11, then the number is divisible by 11.

5th 4th 3rd 2nd 1st
6 4 2 0 7

$7 + 2 + 6 = 15$
$0 + 4 = 4$
Difference 11   64207 is divisible by 11.

To test for divisibility by 7, cross out the last digit. Double the crossed out digit and subtract the result from the remaining number. Continue this process until the number is less than 100. If the last number is a multiple of 7, then the original number is divisible by 7.

$60\cancel{9}$ cross out 9
18 double 9, subtract
42 42 is a multiple of 7

609 is divisible by 7

$5919\cancel{2}$ cross out 2
4 double 2, subtract
$591\cancel{5}$ cross out 5
10 double 5, subtract
$58\cancel{1}$ cross out 1
2 double 1, subtract
56 56 is a multiple of 7

59192 is divisible by 7

DIVISIBILITY

Write yes or no to complete the chart for divisility of the numbers given in the column by the number given across the chart.

Number	2	3	4	5	6	7	8	9	10	11
① 4,950										
② 16,264										
③ 24,948										
④ 98,431										
⑤ 6,259										
⑥ 530,530										

Divisible by-

Answers

divisible by:

	2	3	4	5	6	7	8	9	10	11
①	yes	yes	yes	yes	yes	no	no	yes	yes	yes
②	yes	no	yes	no	no	no	yes	no	no	no
③	yes	yes	yes	no	yes	yes	no	yes	no	yes
④	no	no	no	no	no	no	no	no	no	no
⑤	no	no	no	no	no	no	no	no	no	yes
⑥	yes	no	no	yes	no	yes	no	no	yes	yes

ACTIVITIES

Edited by
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Baltimore County Public Schools
Towson, Maryland
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Upper Montclair, New Jersey

NUMBERS AND MYSTICISM

By William F. Oberle III, Baltimore County Public Schools, Towson, MD 21204

Teacher's Guide

Grade level: 7–12

Materials: One set of worksheets for each student.

Objectives: The student will be able to list all the divisors of a given rational number; determine whether a number is perfect, deficient, or abundant by applying the definition; and determine whether two numbers are amicable by applying the definition.

Discussion: Investigating the mystical connotations attached to numbers is a natural way to generate student interest in number theory. Since the Bible states the earth was created in six days, people felt that six was a mystical number. Likewise, the number twenty-eight was held in esteem because it is the number of days in the lunar cycle. Even in the twentieth century, people have associated the number 666 (the mark of the beast in Revelation) with certain individuals. Before using the following worksheets, the student must be familiar with the concept of a divisor of a number and with the notation below.

$D_n = \{\text{all divisors of } n\}$, $D_8 = \{1, 2, 4, 8\}$, $D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$, $D_{284} = \{1, 2, 4, 71, 142, 284\}$

Answers:

Sheet 1

1. Abundant numbers 1–20: 12, 18, 20
2. Deficient numbers 1–20: 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19
3. 28 is a perfect number: $D_{28} = \{1, 2, 4, 7, 14, 28\}$; $1 + 2 + 4 + 7 + 14 = 28$.
   $D_{496} = \{1, 2, 4, 8, 16, 31, 62, 124, 248, 496\}$; the sum is 496.
4. $D_{1184} = \{1, 2, 4, 6, 16, 32, 37, 74, 148, 296, 592, 1184\}$; the sum is 1210.
   $D_{1210} = \{1, 2, 5, 10, 11, 22, 55, 110, 121, 242, 605, 1210\}$; the sum is 1184.

Sheet 2

1. M-15, I-10, C-4, H-9, A-1, E-6, L-33; total, 78; $D_{78} = \{1, 2, 3, 6, 13, 26, 39, 78\}$; the sum is 90; abundant
2. K-18, A-1, T-81, H-9, R-85, Y-90, N-17; total, 301; $D_{301} = \{1, 7, 43, 301\}$; the sum is 51; deficient
3. N-17, E-6, W-66, Y-90, O-47, R-85, K-18, C-4, I-10, T-81, Y-90; total, 514; $D_{514} = \{1, 2, 257, 524\}$; the sum is 260; deficient
4. B-2, I-10, L-33, L-33, I-10, E-6, J-13, E-6, A-1, N-17, K-18, I-10, N-17, G-8; total, 184; $D_{184} = \{1, 2, 4, 8, 23, 46, 92, 184\}$; the sum is 175; deficient

Sheet 3

1. Fire—total, 108; abundant
   Water—total, 239; deficient
2. Chris Evert—total, 403; deficient
   Jimmy Connors—total, 440; abundant
3. President—total, 354; abundant
   Jimmy Carter—total, 405; deficient

Numbers and Mysticism

Consider $D_{284} = \{1,2,4,71,142,284\}$. The proper divisors of 284 are all the divisors except the number itself. Since $1 + 2 + 4 + 71 + 142 = 220$, the sum of the proper divisors is 220.

What is the sum of the proper divisors of 24?

Now consider the following definitions:

1. A number is called abundant if the sum of its proper divisors exceeds the number.

   Example: 24 is abundant, since
   $1 + 2 + 3 + 4 + 6 + 8 + 12 = 36$.

   Find all the abundant numbers between 1 and 20..

2. A number is called deficient if the sum of its proper divisors is less than the number.

   Example: 8 is deficient, since $1 + 2 + 4 = 7$.

   Find the deficient numbers between 1 and 20.

3. A number is called perfect if the sum of its proper divisors equals the number.

   Example: 6 is perfect, since $1 + 2 + 3 = 6$.

   Find a perfect number between 20 and 30.

   Show that 496 is perfect.

4. Two numbers are amicable if each is the sum of the proper divisors of the other.

   Example: $D_{284} = \{1,2,4,71,142,284\}$, and
   $1 + 2 + 4 + 71 + 142 = 220$.
   $D_{220} = \{1,2,4,5,10,11,20,22,44,55,110,220\}$, and
   $1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$.
   220 and 284 are amicable.

   Show that 1184 and 1210 are amicable.

SHEET 2

Numbers and Mysticism

Much of the mysticism surrounding numbers dates back to the time of Pythagoras (500 B. C.). Special meanings were given to numbers depending on whether they were perfect, abundant, deficient, or amicable. For example, two talismans bearing numbers that are amicable would seal perfect friendship between the wearers. Persons or objects associated with deficient numbers were considered inferior, with abundant numbers above average, and with a perfect number--well, you can imagine. Below you will find a number assigned to each letter of the alphabet. Using this assignment, you can decide whether a person or object is associated with a deficient, abundant, or perfect number.

A - 1
B - 2
C - 4
D - 5
E - 6
F - 7
G - 8
H - 9
I - 10
J - 13
K - 18
L - 33
M - 15
N - 17
O - 47
P - 64
Q - 29
R - 85
S - 80
T - 81
U - 14
V - 37
W - 66
X - 87
Y - 90
Z - 100

Example: My first name, William

W - 66
I - 10
L - 33
L - 33
I - 10
A - 1
M - 15
168

$D_{168} = \{1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168\}$

$1+2+3+4+6+7+8+12+14+21+24+28+42+56+84 = 312$

The number is abundant since the sum of the proper divisors (312) is greater than the number (168).

Use the procedure in the example to analyze the following:

1. The name Michael
2. The name Kathryn
3. New York City
4. Billie Jean King
5. Your school's name
6. Your teacher's name

Numbers and Mysticism

To compare two persons or objects, use the following rules:

1. Amicable numbers indicate a perfect relationship.
2. If the numbers are both abundant or both deficient, a good relationship exists.
3. If one number is abundant and the other deficient, a poor relationship exists.

Example: School and student

S - 80	S - 80
C - 4	T - 81
H - 9	U - 14
O - 47	D - 5
O - 47	E - 6
L - 33	N - 17
220	T - 81
	284

On sheet 1, the numbers 220 and 284 were shown to be amicable - no wonder they get along so well together!

Try the following:

1. Fire and water
2. Chris Evert and Jimmy Connors
3. The word President and the name Jimmy Carter
4. Your name and your desired profession

The way the letters of the alphabet are assigned numbers is arbitrary. Try your hand at making up an assignment so that you and a friend will be amicable (220 and 284).

A -
B -
C -
D -
E -
F -
G -
H -
I -
J -
K -
L -
M -
N -
O -
P -
Q -
R -
S -
T -
U -
V -
W -
X -
Y -
Z -

activities

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PATTERN GAZING

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Teacher's Guide

Grade Levels: 7–12

Materials: One set of worksheets for each student, a transparency of each worksheet, calculators (optional).

Objectives: To enable the students to discover patterns and make generalizations.

Directions: Distribute the activity sheets one at a time. With each sheet, discuss with the students the information presented in the charts. Be sure they understand that the three dots (. . .) represent unspecified intermediate addends. You may need to discuss how to determine the number of addends N without counting each term. Ask the students to examine the relationship between the last number in the series and N of that series. After the students have completed the tables, encourage them to seek a relationship between N and the sum S. The two series following the table are given to test the student's ability to apply the generalization.

Your students may experience some difficulty in determining N for the first three activities. If so, consider discussing the relationship between the number of terms in the given series with the number of terms in the series of counting numbers. For example, compare the number of terms in the series $1 + 3 + 5$ with $1 + 2 + 3 + 4 + 5 + 6$. Students should recognize that the second series has six terms and can be regrouped as $(1 + 3 + 5) + (2 + 4 + 6)$. Since there are as many odd-number terms as even-number terms, the number of odd-number terms will always be half the number of the terms in the second series. Additional examples of this type may be necessary. This generalization will prove helpful in determining the sum of the series following each table.

After each table, the students are asked to write their rule in words or symbols. For most students, a statement such as "multiply N times itself" for (1) or "multiply N times $(N + 1)$" for (3) should suffice. Advanced algebra students, however, might be expected to generalize in (3), for example, that $2 + 4 + 6 + \cdots + (2n) = n(n + 1)$.

After the students have completed all the tables, ask them to look back at their results. Encourage the students to see how their rules for the various tables relate to each other. In activity 4, the rule is

$$S = \frac{N(N + 1)}{2}.$$

The rule for activity 6 is

This section is designed to provide mathematical activities suitable for reproduction in worksheet and transparency form for classroom use. Appropriate laboratory experiences, discovery activities, and model constructions of mathematical significance drawn from the topics of the seventh, eighth, and nine grades are most welcome. Send your suggestions, in a four page format similar to the "Activities" already published, to the managing editor for review.

PATTERN GAZING

(1) Complete this table to see what happens when we add odd numbers.

Series	No. of Addends (N)	Sum (S)
1	1	1
1 + 3	2	4
1 + 3 + 5		
1 + 3 + 5 + 7		
1 + 3 + 5 + 7 + 9		
1 + 3 + 5 + 7 + 9 + 11		

Can you find a way to calculate the sum without adding each term? If so, use your rule to calculate these sums:

$1 + 3 + \cdots + 17 + 19 =$ ______________________________

$1 + 3 + \cdots + 23 + 25 =$ ______________________________

Write your rule in words or symbols. ______________________

____

(2) Let's see what happens when we alternately add and subtract odd numbers. Complete the table.

Series	No. of Addends (N)	Sum (S)
1	1	1
1 - 3	2	-2
1 - 3 + 5		
1 - 3 + 5 - 7		
1 - 3 + 5 - 7 + 9		

Can you find a way to calculate the sum without alternately adding and subtracting each term? If so, use your rule to calculate these sums:

$1 - 3 + \cdots + 17 - 19 =$ ______________________________

$1 - 3 + \cdots + 41 - 43 =$ ______________________________

Write your rule in words or symbols. ______________________

____

____

PATTERN GAZING

(3) Let's try adding even numbers.

Series	No. of Addends (N)	Sum (S)
2	1	2
2 + 4	2	6
2 + 4 + 6		
2 + 4 + 6 + 8		
2 + 4 + 6 + 8 + 10		
2 + 4 + 6 + 8 + 10 + 12		

Can you find a way to calculate the sum without adding each term? If so, use your rule to calculate these sums:

2 + 4 + 6 + ··· + 26 + 28 = ____________________

2 + 4 + 6 + ··· + 48 + 50 = ____________________

Write your rule in words or symbols.____________________

____

(4). Let's see what happens when we add both odd and even numbers.

Series	No. of Addends (N)	Sum (S)
1	1	1
1 + 2	2	3
1 + 2 + 3		
1 + 2 + 3 + 4		
1 + 2 + 3 + 4 + 5		

Can you find a way to calculate the sum without adding each term? (Hint: Compare chart 4 with chart 3)

If so, use your rule to calculate these sums:

1 + 2 + ··· + 17 + 18 = ____________________

1 + 2 + ··· + 43 + 44 = ____________________

Write your rule in words or symbols.____________________

____

____

SHEET 3

PATTERN GAZING

(5) Let's see what happens when we square first and then alternately add and subtract.

Series	No. of Addends (N)	Sum (S)
1^2	1	1
$1^2 - 2^2$	2	-3
$1^2 - 2^2 + 3^2$		
$1^2 - 2^2 + 3^2 - 4^2$		
$1^2 - 2^2 + 3^2 - 4^2 + 5^2$		

Can you find a way to calculate the sum without alternately adding and subtracting each term? (HINT: Compare chart 5 with chart 4)

If so, use your rule to calculate these sums:

$1^2 - 2^2 + \cdots + 17^2 - 18^2 =$ ______________________________

$1^2 - 2^2 + \cdots - 24^2 + 25^2 =$ ______________________________

Write your rule in words or symbols.____________________

____

(6) What about adding the cubes of the counting numbers? Complete the table.

Series	No. of Addends (N)	Sum (S)
1^3	1	1
$1^3 + 2^3$	2	9
$1^3 + 2^3 + 3^3$		
$1^3 + 2^3 + 3^3 + 4^3$		
$1^3 + 2^3 + 3^3 + 4^3 + 5^3$		

Can you find a way to calculate the sum without adding each term? (HINT: Compare chart 6 with chart 4).

If so, use your rule to calculate these sums:

$1^3 + 2^3 + \cdots + 13^3 + 14^3$?______________________________

$1^3 + 2^3 + \cdots + 19^3 + 20^3$?______________________________

Write your rule in words or symbols.____________________

____

$$S = \left[\frac{N(N+1)}{2}\right]^2.$$

This latter result says that the sum of the first N cubes is the square of the sum of the first N natural numbers.

Most students should be able to substantiate the relationship between charts 3 and 4. In chart 3, $2 + 4 + 6 + \cdots + (2n) = n(n+1)$, and in chart 4, $1 + 2 + 3 + \cdots + n = n(n+1)/2$. A statement such as "each term in the series in chart 4 is half the corresponding term in the parallel series in chart 3" should suffice. Students familiar with factoring may see the relationship as $2 + 4 + 6 + \cdots + (2n) = 2(1 + 2 + 3 + \cdots + n)$.

Follow-up Activities: Examine these series for similar patterns:

(1) $2 - 4 + 6 - 8 + \cdots$
(2) $1 - 2 + 3 - 4 + \cdots$
(3) $1^2 + 2^2 + 3^2 + \cdots$

Answers:

1. $10^2 = 100$; $13^2 = 169$; $s = n^2$
2. -10; -22; $S = \begin{cases} N \text{ if } N \text{ is odd} \\ -N \text{ if } N \text{ is even} \end{cases}$
3. $(14)(15) = 210$; $(25)(26) = 650$;
   $S = N(N+1)$
4. $\dfrac{(18)(19)}{2} = 171$; $\dfrac{(44)(45)}{2} = 990$;
   $S = \dfrac{N(N+1)}{2}$
5. -171; 325;
   $S = \begin{cases} N(N+1)/2 \text{ if } N \text{ is odd} \\ -N(N+1)/2 \text{ if } N \text{ is even} \end{cases}$
6. $\left[\dfrac{(14)(15)}{2}\right]^2 = 11025$;
   $\left[\dfrac{(20)(21)}{2}\right]^2 = 44100$;
   $S = \left[\dfrac{N(N+1)}{2}\right]^2$

AN APPLICATION OF AN ALGEBRAIC PROOF

Which do you prefer? Sales tax, then discount, or discount, then sales tax?

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We recently purchased a book from our campus book store. As a faculty member, one of us was eligible for a discount. We thought we were getting a good deal when the cashier added the sales tax to the item purchased and then applied the discount to the entire sum. On closer inspection, however, it became evident that it would have made no difference in the final amount if the cashier had reversed the process—that is, applied the deduction first and then taxed the difference.

In talking to friends, we learned that it was not intuitively obvious to many that the order of applying tax and discount made no difference. In fact, only by the use of a numerical illustration were we able to convince some of our acquaintances that the order of application made no difference. For example:

Retail price of books	\$100
Sales tax	6%
Discount	10%

Tax first, then discount, where C is cost:

$$(\$100 + 6\%\text{ tax}) - 10\%\text{ disc.} = C$$
$$[\$100 + 0.06\,(\$100)] - 0.1\,[\$100 + 0.06\,(\$100)] = C$$
$$\$106.00 - \$10.60 = \$95.40$$

Discount first, then tax:

$$(\$100 - 10\%\text{ disc.}) + 6\%\text{ tax} = C$$
$$[\$100 - 0.1\,(\$100)] + 0.06\,[\$100 - 0.1\,(\$100)] = C$$
$$\$90.00 + 5.40 = \$95.40$$

This type of numerical illustration is not sufficient for mathematics classes, so an algebraic argument follows:

Let p be the price of book,
t be the tax,
d be the discount.

Tax first, then discount = discount first, then tax:

$$(p + tp) - d(p + tp) = (p - dp) + t(p - dp)$$
$$p + tp - dp - dtp = p - dp + tp - tdp$$
$$p + tp - dp - dtp = p + tp - dp - tdp$$
$$0 = 0$$

This proof enables the student not only to prove that the order of tax and discount makes no difference in cost, but also to predict that it is true for any combination of sales tax and discount.

As the problem is examined more closely, one will find that the order of taxing and discounting does make a difference to the store proprietor. Using the previous illustration, a 6% tax followed by a 10% discount, the store would lose to the state in sales taxes approximately 0.6% of the total sale, whereas in the opposite order it would be 0.6% to the store's advantage

Tax first, then deduction:

6% tax on \$100 is \$6.00

Deduction first, then tax:

6% tax on \$90 is \$5.40
\$0.60 on \$100 is 0.6%

In stores with meager profits, this small percentage could be significant.

Also, students typically find this process much more confusing and difficult to carry out than the procedure for the case $a = 1$, particularly when one or more of a, b, and c is negative.

The purpose of this discussion is to point out that, like the method for factoring $x^2 + bx + c$, factoring $ax^2 + bx + c$ can also be made to depend on factoring an integer (in this case ac) in such a way that the algebraic sum of the factors is b. Since this is exactly what was needed in the easier case when the leading coefficient was 1, nothing new must be learned or taught, and the transition to the more general case is seen as natural and easy by the students. We acknowledge that the method discussed here is not original with the authors. At the same time, it is not widely known in this country, and we feel that it should be, since our experience indicates that it is much more easily and efficiently taught.

A second purpose of this discussion is to show how a careful but easy examination of the prime factors of a and c can often greatly reduce the possible factorings of ac that must be considered in factoring $ax^2 + bx + c$. With this added touch, we call the method the "primes breakdown" method of factoring.

Factoring $ax^2 + bx + c$

As noted above, factoring $ax^2 + bx + c$ requires finding integers A, B, C, and D such that

$$AC = a,\ AD + BC = b, \text{ and } BD = c.$$

Let $h = AD$ and $k = BC$. Then

$$hk = AD \cdot BC = AC \cdot BD = ac,$$

and the process immediately becomes one of finding h and k whose product is ac and whose algebraic sum is b. If this can be done, we have

$$\begin{aligned} ax^2 + bx + c &= \frac{hk}{c}x^2 + (h + k)x + c \\ &= \frac{1}{c}\{hkx^2 + hcx + kcx + c^2\} \\ &= \frac{1}{c}\{hx(kx + c) + c(kx + c)\} \\ &= \frac{1}{c}(hx + c)(kx + c), \end{aligned}$$

and this easily reduces to a product of linear factors with integral coefficients, since c evenly divides both $hk = ac$ and c.

Example 1. Factor $6x^2 + 19x + 10$.

Solution. We seek integers h and k such that $hk = 60$ and $h + k = 19$. Clearly, 4 and 15 do the job, and we can write

$$\begin{aligned} 6x^2 + 19x + 10 &= 6x^2 + 4x + 15x + 10 \\ &= 2x(3x + 2) + 5(3x + 2) \\ &= (2x + 5)(3x + 2). \end{aligned}$$

Note that it makes no difference whether we choose 4 or 15 to be h or k. Thus, the equation above could just as well have been written

$$\begin{aligned} 6x^2 + 19x + 10 &= 6x^2 + 15x + 4x + 10 \\ &= 3x(2x + 5) + 2(2x + 5) \\ &= (3x + 2)(2x + 5). \end{aligned}$$

Example 2. Factor $9x^2 - 9x - 10$.

Solution. Here we seek h and k such that $hk = {}^-90$ and $h + k = {}^-9$. Clearly, one of h and k must be negative and the other positive, and it is not difficult to see that 6 and $^-15$ do the job. Thus, we can write

$$\begin{aligned} 9x^2 - 9x - 10 &= 9x^2 + 6x - 15x - 10 \\ &= 3x(3x + 2) - 5(3x + 2) \\ &= (3x - 5)(3x + 2). \end{aligned}$$

We acknowledge that some students may not be able to factor as swiftly using this method as they would using the usual approach. However, quickness is surely not a serious goal, and any loss of speed is more than compensated for by the ease in teaching and the reduction in student confusion.

Primes Breakdown Method

Factoring quadratic polynomials can often be made even easier by writing $hk = ac$ as a product of prime powers and noting that a prime that divides hk and divides h also divides $h + k$ if and only if it also divides k. The idea is easily illustrated by looking again at the examples considered above. Thus, in factoring $6x^2 + 19x + 10$ of example 1, we seek h and k such that $hk = 6 \cdot 10 = 2^2 \cdot 3 \cdot 5$ and $h + k = 19$. Since 2 does not divide 19, it follows that 2 does not divide both h and k. Thus, the possible factorings can only be 4×15, 12×15, $3 \times$

20, or 1 × 60. Since $4 + 15 = 19$, we choose 4 and 15 as h and k, and the process now goes on as above.

To factor the quadratic $9x^2 - 9x - 10$ of example 2, we seek h and k such that $hk = {}^{-}90 = {}^{-}(3)^2 \cdot 2 \cdot 5$ and $h + k = {}^{-}9$. Since 3 divides $^{-}9$, 3 must divide both h and k, and except for the sign, the only possible factorings are 3×30 and 6×15. Since $6 + ({}^{-}15) = {}^{-}9$, we choose 6 and $^{-}15$ as h and k and proceed as before.

For small values of the product ac, the primes-breakdown method may not effect much saving, but for large values it is often quite helpful. Thus, as a final example, consider

$$x^2 + 19x - 216.$$

Since $hk = {}^{-}216 = {}^{-}3^3 \cdot 2^3$ and neither 2 nor 3 divides 19, it follows that neither 2 nor 3 divides both h and k. Thus, apart from the sign, the only possible factorings are 1×216 and 27×8. Since $19 = 27 + (-8)$, the factoring is easily completed, and we have

$$x^2 + 19x - 216 = (x + 27)(x - 8)$$

as desired.

In our experience, the use of the primes-breakdown method of factoring has removed most of the guesswork associated with the traditional trial-and-error method of factoring. The results are that less time is required for most students to achieve good comprehension and that the entire subject is more easily taught. The authors encourage other teachers to use this method and invite correspondence from readers regarding its use.

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Mathematics Objective

13. Write a number as the product of its prime factors express prime factorization using exponents

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Prime factorization For teaching suggestions see page 204 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman) For teaching suggestions see page 77-78 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	For teaching suggestions see page 59 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) 7 practice p. 70 7 HA p. 21 For teaching suggestions see page 66-67 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) In Guide: Prime Factorization (notes) Prime Factorization (worksheet) ans

PRIME FACTORIZATION

A prime number is a positive whole number greater than one that has exactly two different factors.

$5 = 5 \cdot 1 \qquad 11 = 11 \cdot 1 \qquad 23 = 23 \cdot 1$

One is not a prime number because it has only one factor, itself.

A composite number is a positive number that has more than two different factors. Every composite number can be expressed as the product of its prime factors. This is called the prime factorization of the number.

One method used in finding the prime factorization of a composite number is successive division by prime numbers. Use divisibility rules to determine divisors:

$$\begin{array}{r|l} 2 & 216 \\ \hline 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

The unit digit is an even number, $\therefore$ divisible by 2.

The sum of the digits is a multiple of three, $\therefore$ divisible by 3.

Continue division until the dividend is a prime number.

$$216 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$$
$$= 2^3 \cdot 3^3$$

The product of all divisors and the prime number resulting from the last step is the prime factorization of the number.

Prime Factorization

Show the prime factorization of each of the following using exponents:

① $756 =$

② $864 =$

③ $384 =$

④ $1250 =$

⑤ $2028 =$

⑥ $288 =$

Ans: ① $2^2 \cdot 3^3 \cdot 7$

② $2^5 \cdot 3^3$

③ $2^7 \cdot 3$

④ $2 \cdot 5^4$

⑤ $2^5 \cdot 3^2$

Mathematics Objective

15. Find the greatest common factor and the least common multiple of two numbers.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
GCF LCM For teaching suggestions see page 206-207 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman) For teaching suggestion see page 79-80 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	7-H 22 Euclidean algorithm for finding G.C.F. For teaching suggestions see page 66-67 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) p. 67 historical note: Euclidean algorithm for finding g.c.f. MRC: MS C30, C31, C40, C41, C42 In Guide: Greatest common factor Least common mutliple Factors and multiples answers

Greatest Common Factor

A factor is a divisor. The greatest positive integer that is a common factor of two or more integers is called their greatest common factor or GCF.

A method for finding GCF	18, 24, 30
1. Express each integer in prime factorization	$18 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$ $24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$ $30 = 2 \cdot 3 \cdot 5 = 2 \cdot 3 \cdot 5$
2. Select the least power of each common factor.	$2 \cdot 3$
3. Find the product of these powers.	$2 \cdot 3 = 6$
4. The product is the GCF	The GCF of 18, 24, and 30 is 6.

Find the GCF of the following:

① 39 =
52 =
65 =
GCF =

② 28 =
42 =
GCF =

③ 42 =
56 =
72 =
GCF =

④ 18 =
39 =
45 =
GCF =

⑤ 18 =
24 =
42 =
GCF =

⑥ 45 =
54 =
GCF =

Least Common Multiple

A multiple will be a product. The least common multiple of two or more integers will be the smallest multiple that each of the integers is a factor of or will divide into. LCM.

A method for finding LCM 12, 27, 30

1. Express each integer in prime factorization form	$12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$ $27 = 3 \cdot 3 \cdot 3 = 3^3$ $30 = 2 \cdot 3 \cdot 5$
2. Select the greatest power of each prime factor.	2^2, 3^3, and 5
3. Find the product of these powers	$2^2 \cdot 3^3 \cdot 5 = 4 \cdot 27 \cdot 5 = 540$
4. This product is the LCM	The LCM of 12, 27, and 30 is 540.

Find the LCM of the following:

① 11 =
14 =
22 =
LCM =

② 8 =
16 =
20 =
LCM =

③ 14 =
5 =
10 =
LCM =

④ 3 =
12 =
8 =
LCM =

Factors and Multiples

Factor each of the following in prime factorization with exponents:

① 168 ② 324 ③ 3648 ④ 408

Find the value of n in the following:

⑤ $(n)(7)(11) = 924$ ⑦ $(3)^n(7) = 189$ ⑨ $(2)^3(n)^2 = 200$

⑥ $(2)^n(3)^n = 1296$ ⑧ $(3)^2(5)^n = 1125$ ⑩ $(2)^4(3)^2(17) = n$

Find the GCF and LCM for each set of numbers:

⑪ 16
24
40
GCF =
LCM =

⑫ 9
15
21
GCF =
LCM =

⑬ 16
5
20
GCF =
LCM =

⑭ 12
15
18
GCF =
LCM =

⑮ 21
30
54
45
GCF =
LCM =

Answers

Greatest Common Factor

① 13
② 14
③ 2
④ 3
⑤ 6
⑥ 9

Least Common Multiple

① 308
② 160
③ 70
④ 24

Factors and Multiples

① $2^2 \cdot 3 \cdot 7$
② $2^2 \cdot 3^4$
③ $2^6 \cdot 3 \cdot 19$
④ $2^3 \cdot 3 \cdot 17$
⑤ 12
⑥ $2^4 \cdot 3^4$
⑦ 3^3
⑧ 5^3
⑨ 5^2
⑩ 2448
⑪ GCF = 8
LCM = 240
⑫ GCF = 3
LCM = 315
⑬ GCF = 1
LCM = 80
⑭ GCF = 3
LCM = 180
⑮ GCF = 3
LCM = 1890

Mathematics Objective

15M Read and write fractions and mixed numbers.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Mixed numeral - meaning, as quotients For teaching suggestions see page 213, 218 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman) For teaching suggestions see page 109-110 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	For teaching suggestions see page 135-137 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) 7 Practice - L 42, 43 8 Practice p. 61 8-L-32 For teaching suggestions see page 77 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) Skillmaster 8-17 Skillmaster 7-20 MRC: AWA I p. 6 MS - D 41

Mathematics Objective

15M Rename fractions in lowest terms using g.c.f. and complete missing parts of equivalent fractions

Teaching Techniques and Introductory Activities

Renaming fractions in lowest terms
For teaching suggestions see page 216-17 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

p. 115
7 Mathematics for Mastery
(Silver Burdett)

Lowest terms
For teaching suggestions see page 115 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Missing parts of equal fractions
For teaching suggestions see page 112-113 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)

Compare fractional numbers
For teaching suggestions see page 111, 116 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Supplementary Activities and Resources

For teaching suggestions see page 132-134 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

7 Practice p. 73, 74
8 Practice p. 59. 60

p. 80
8 (Silver Burdett)

MRC:
AWAI p. 6, 12
MS-D12, D13

In Guide: Fractions - simplest form
(answers)

Equivalent Fractions
(answers)

Lowest terms
For teaching suggestions see page 80 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

Missing parts of equal fractions
For teaching suggestions see page 74-75 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)
p. 88 (1-4)

Comparing fractional numbers
For teaching suggestions see page 79 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)
p. 88 (5-8)
Skillbook 8-17

Fractions - Simplest Form

Equivalent fractions represent the same rational number.

Two integers are relatively prime when their greatest common factor is one.

A fraction is in simplest form when the numerator and denominator are relatively prime.

Any number divided by itself is one. The Property of One states that the product of one and any number is that number. This property is used when renaming fractions in simplest form.

$$\frac{30}{105} = \frac{2\cdot3\cdot5}{7\cdot3\cdot5} = \frac{2}{7}\cdot\left[\frac{3\cdot5}{3\cdot5}\right] = \frac{2}{7}$$

$$\frac{9a^2b}{15ab^2} = \frac{3\cdot3\cdot a\cdot a\cdot b}{3\cdot5\cdot a\cdot b\cdot b} = \frac{3\cdot a}{5\cdot b}\cdot\left[\frac{3\cdot a\cdot b}{3\cdot a\cdot b}\right] = \frac{3a}{5b}$$

Use this method to simplify the following:

① $\frac{28}{52} =$

② $\frac{36}{75} =$

③ $\frac{525}{695} =$

④ $\frac{48}{72} =$

⑤ $\frac{432}{2520} =$

⑥ $\frac{6a^2}{12a} =$

⑦ $\frac{27ab}{81ab} =$

⑧ $\frac{108a^4}{144a^3} =$

⑨ $\frac{40a^3x^4}{60a^2x^5} =$

⑩ $\frac{7abx}{21acx} =$

Equivalent Fractions

Find the missing part in each of the following:

① $\frac{3}{5} = \frac{12}{\quad}$ ② $\frac{4}{\quad} = \frac{7}{35}$ ③ $\frac{\quad}{9} = \frac{5}{15}$

④ $\frac{9}{24} = \frac{\quad}{56}$ ⑤ $\frac{20}{30} = \frac{\quad}{12}$ ⑥ $\frac{13}{11} = \frac{\quad}{33}$

⑦ $\frac{24}{27} = \frac{\quad}{45}$ ⑧ $\frac{35}{40} = \frac{14}{\quad}$ ⑨ $\frac{24}{\quad} = \frac{42}{77}$

⑩ $\frac{36}{60} = \frac{\quad}{55}$ ⑪ $\frac{\quad}{36} = \frac{85}{90}$ ⑫ $\frac{26}{\quad} = \frac{117}{135}$

⑬ $\frac{6a}{7b} = \frac{\quad}{21ab^2}$ ⑭ $\frac{9a^2b}{10c} = \frac{\quad}{20ac}$ ⑮ $\frac{\quad}{4b} = \frac{10a}{5b}$

Simplify the following:

⑯ $\frac{125}{250} =$ ⑰ $\frac{144}{256} =$ ⑱ $\frac{1312}{1476} =$

⑲ $\frac{9a^2}{27a^3b} =$ ⑳ $\frac{28a^4}{56a^3} =$ ㉑ $\frac{72a^2b^2}{128ab} =$

Mathematics Objective

15 Compare fractions and mixed numbers using the symbols >, <, or =

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Comparing fractions For teaching suggestions see page 224-25 of the Teacher's Edition of *Mathematic Around Us* 7 (Scott Foresman)	For teaching suggestions see page 157 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)
To compare fractional numbers p. 116-117 7 (Silver Burdett)	7-H-23 8 practice p. 70
	To compare fractional numbers p. 79 8 (Silver Burdett)
	In Guide: Fractions (ans)

FRACTIONS

Use the symbol >, <, or = to complete the following:

① $\frac{1}{4}$ ○ $\frac{5}{16}$ ② $\frac{4}{5}$ ○ $\frac{13}{15}$ ③ $\frac{5}{7}$ ○ $\frac{9}{14}$

④ $\frac{5}{9}$ ○ $\frac{13}{18}$ ⑤ $\frac{9}{12}$ ○ $\frac{15}{24}$ ⑥ $\frac{3}{4}$ ○ $\frac{2}{3}$

⑦ $\frac{9}{16}$ ○ $\frac{4}{7}$ ⑧ $\frac{10}{11}$ ○ $\frac{11}{12}$ ⑨ $\frac{13}{19}$ ○ $\frac{5}{7}$

Arrange in order from least to greatest

⑩ $\frac{3}{4}, \frac{7}{8}, \frac{2}{3}, \frac{1}{2}$ ⑪ $\frac{5}{6}, \frac{2}{3}, \frac{17}{19}, \frac{3}{4}, \frac{11}{15}$

Which answer is not equivalent to the given fraction

⑫ $\frac{14}{20}$ ⓐ $\frac{42}{60}$ ⓑ $\frac{20}{30}$ ⓒ $\frac{28}{40}$ ⓓ $\frac{7}{10}$

⑬ $\frac{7}{8}$ ⓐ $\frac{35}{40}$ ⓑ $\frac{28}{32}$ ⓒ $\frac{14}{16}$ ⓓ $\frac{21}{25}$

⑭ $\frac{3}{7}$ ⓐ $\frac{6}{16}$ ⓑ $\frac{12}{28}$ ⓒ $\frac{18}{42}$ ⓓ $\frac{45}{105}$

⑮ $\frac{3}{12}$ ⓐ $\frac{3+3}{12+12}$ ⓑ $\frac{3 \times 4}{3 \times 16}$ ⓒ $\frac{4}{16} \times \frac{7}{7}$ ⓓ $\frac{4}{10} \times \frac{3}{4}$

⑯ $\frac{4}{6}$ ⓐ $\frac{6}{9} \times \frac{2}{3}$ ⓑ $\frac{4}{15} \cdot \frac{15}{6}$ ⓒ $\frac{2}{3} \cdot \frac{14}{14}$ ⓓ $\frac{18}{27}$

⑰ $\frac{3}{8}$ ⓐ $\frac{18}{48} \div \frac{3}{3}$ ⓑ $\frac{24 \div 4}{64 \div 4}$ ⓒ $\frac{9 \cdot 5}{24 \cdot 5}$ ⓓ $\frac{12 \cdot 2}{32 \cdot 3}$

⑱ $\frac{2}{6}$ ⓐ $\frac{12}{36}$ ⓑ $\frac{30}{80}$ ⓒ $\frac{42 \div 7}{36 \div 2}$ ⓓ $\frac{9 \cdot 4}{6 \cdot 18}$

⑲ $\frac{14}{18}$ ⓐ $\frac{28}{36}$ ⓑ $\frac{14 \div 2}{18 \div 2}$ ⓒ $\frac{42}{54}$ ⓓ $\frac{14 \cdot 6}{9 \cdot 9}$

⑳ $\frac{2}{5}$ ⓐ $\frac{4}{10}$ ⓑ $\frac{14}{30}$ ⓒ $\frac{8}{20}$ ⓓ $\frac{18}{45}$

Mathematics Objective

16M Add and subtract fractions and mixed numbers.

Teaching Techniques and Introductory Activities

Adding fractions and mixed numbers
For teaching suggestions see page 226-29 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Subtracting fractions and mixed numbers.
For teaching suggestions see page 232-35 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Adding fractions and mixed numerals
For teaching suggestions see page 126 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Subtracting fractions and mixed numerals
For teaching suggestions see page 127-8 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Supplementary Activities and Resources

Adding fractions
For teaching suggestions see page 158-161 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

8-practice p. 72 8 L-37 8 H-17

For teaching suggestions see page of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

7 practice p. 83-84
7 L-48, H-24

Adding and subtracting with mixed numerals
For teaching suggestions see page 81-88 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

7 Skillmaster 7-24, 7-25
8 Skillmaster 8-20, 8-21

In Guide: Fractions-Addition and
Subtraction
ans.

MRC:
AWA: (I) 16, 17, 20, 21
MS - D-32, D-34

FRACTIONS - ADDITION and SUBTRACTION

① $\frac{3}{8} + \frac{2}{3} =$

② $\frac{3}{5} + \frac{1}{6} =$

③ $\frac{5}{8} + \frac{1}{2} + \frac{1}{12} =$

④ $\frac{5}{8} + \frac{3}{5} + \frac{9}{10} =$

⑤ $\frac{4}{5} + \frac{1}{2} + \frac{3}{4} + \frac{7}{10} + \frac{1}{20} =$

⑥ $5\frac{2}{3} + 1\frac{5}{6} =$

⑦ $8\frac{7}{8} - 2\frac{3}{4} =$

⑧ $1\frac{13}{15} + 2\frac{4}{5} + 7\frac{2}{3} =$

⑨ $\left(4\frac{2}{3} + 3\frac{1}{4}\right) - 2\frac{7}{8} =$

⑩ $\left(5\frac{3}{8} - 2\frac{3}{16}\right) + 12\frac{1}{2} =$

⑪ $3\frac{5}{6} + \frac{4}{9} + 1\frac{1}{2} =$

⑫ $8 - \frac{2}{3} =$

⑬ $10\frac{3}{9} - 3\frac{7}{9} =$

⑭ $\left(1\frac{9}{10} + 4\frac{7}{8}\right) - \left(1\frac{3}{5} + 2\frac{1}{4}\right) =$

⑮ $\left(\frac{8}{9} - \frac{7}{18}\right) + \left(\frac{4}{7} - \frac{1}{4}\right) =$

Mathematics Objective

16M multiply fractions and mixed numbers
16. find the reciprocal of a given fraction or mixed number
17. divide fractions and mixed numbers

Teaching Techniques and Introductory Activities

Multiplying fractions and mixed numbers
For teaching suggestions see page 240-42 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Reciprocals
For teaching suggestions see page 243 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Dividing fractions and mixed numbers
For teaching suggestions see page 246-249 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Multiplying fractions and mixed numbers
For teaching suggestions see page 164, 167 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)

p. 163 Calculate for enrichment

Supplementary Activities and Resources

Multiplying fractions and mixed numbers
For teaching suggestions see page 140-144 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

7 practice p. 87   8 practice p. 64
7 L-49   8 L-33

Reciprocals
For teaching suggestions see page 145 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

For teaching suggestions see page 146-148 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

7 practice p. 90-91   7-L-50
8 practice p. 66-67   8-L-34

Multiplying fractions and mixed numbers
For teaching suggestions see page 91, 93-4 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)

Calculate p. 94 for enrichment
Skillbook 8-22
Skillbook 7-33

Mathematics Objective

Continued:

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Reciprocals For teaching suggestions see page 168-9 of the Teacher's Edtion of *Mathematics for Mastery* 7 (Silver Burdett)	Reciprocals For teaching suggestions see page 92 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)
Dividing fractions and mixed numerals For teaching suggestions see page 170-172 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	Dividing fractions and mixed numerals For teaching suggestions see page 93-96 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)
	Skillbook 8-24, 25 Skillbook 7-35, 36
	In Guide: Fractions and Mixed Numerals (ans)
	MRC. AWA I - 24-44 MS D 42, 43, 46, 47

FRACTIONS and MIXED NUMERALS

Complete the following. Watch operation symbols.

① $\frac{3}{5} + \frac{1}{4} =$

② $17\frac{1}{2} + 9\frac{3}{4} + 4\frac{1}{6} =$

③ $11 - 7\frac{2}{3} =$

④ $\left(\frac{7}{8} - \frac{1}{12}\right) + 2\frac{1}{2} =$

⑤ $\left(9\frac{1}{2} - 7\frac{3}{5}\right) + 2\frac{1}{4} =$

⑥ $\frac{1}{3} \div \frac{3}{4} =$

⑦ $2\frac{1}{2} \div 3\frac{1}{4} =$

⑧ $12\frac{3}{4} \cdot 2\frac{1}{6} =$

⑨ $\frac{3}{4} \cdot \frac{5}{2} \cdot \frac{8}{9} =$

⑩ $\left(\frac{1}{2} \cdot \frac{3}{4}\right) \div \frac{1}{8} =$

⑪ $\left(\frac{6}{7} - \frac{9}{14}\right) - \left(\frac{11}{16} - \frac{10}{16}\right) =$

⑫ $\frac{6}{25} \div \frac{7}{10} \times \frac{5}{6} =$

⑬ $5\frac{3}{4} \div 7\frac{1}{8} \div 1\frac{1}{3} =$

⑭ $\left(2\frac{1}{2} \cdot 3\frac{1}{3}\right) \div \left(7 \cdot 6\frac{2}{7}\right) =$

⑮ $3\frac{7}{8} \cdot 5\frac{1}{2} \cdot \frac{1}{3} =$

⑯ $8\frac{3}{5} - 4\frac{3}{4} =$

⑰ $\frac{16}{18} \cdot \frac{2}{3} \cdot \frac{9}{8} \cdot \frac{6}{27} =$

⑱ $\left(\frac{1}{4} \cdot \frac{3}{7}\right) \cdot \frac{8}{11} =$

⑲ $\frac{a^3}{b} \cdot \frac{b^2}{a^2} =$

⑳ $\frac{3a}{4b} \cdot \frac{4b}{3a} =$

㉑ $\frac{5a}{b} \cdot \frac{b}{15c} =$

㉒ $\frac{a^2}{3} \cdot \frac{b^2}{4}$

㉓ $\frac{a}{b} \cdot \frac{c}{a} \cdot \frac{b}{c} =$

Challenge:

$$\frac{\left(\frac{1}{2} + \frac{3}{4}\right)}{1\frac{2}{3} - \frac{7}{8}} \div \frac{\left(5\frac{1}{2} - 4\frac{6}{7}\right)}{\frac{2}{9} \cdot \frac{1}{8}}$$

Answers

Fractions - Simplest Form

① $\frac{7}{13}$
② $\frac{12}{25}$
③ $\frac{105}{139}$
④ $\frac{2}{3}$
⑤ $\frac{6}{35}$
⑥ $\frac{a}{2a}$
⑦ $\frac{1}{3}$
⑧ $\frac{3a}{4}$
⑨ $\frac{2a}{3d}$
⑩ $\frac{b}{3c}$

Equivalent Fractions

① 20
② 20
③ 3
④ 21
⑤ 8
⑥ 39
⑦ 40
⑧ 16
⑨ 44
⑩ 33
⑪ 34
⑫ 30
⑬ $18a^2b$
⑭ $18a^3b$
⑮ $8a$
⑯ $\frac{1}{2}$
⑰ $\frac{9}{16}$
⑱ $\frac{41}{123}$
⑲ $\frac{1}{3ab}$
⑳ $\frac{a}{2}$
㉑ $\frac{9ab}{16}$

Fractions

① $<$
② $<$
③ $>$
④ $<$
⑤ $>$
⑥ $>$
⑦ $<$
⑧ $<$
⑨ $<$
⑩ $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{7}{8}$
⑪ $\frac{2}{3}, \frac{11}{15}, \frac{3}{4}, \frac{5}{6}, \frac{17}{19}$
⑫ $\frac{20}{30}$
⑬ $\frac{21}{25}$
⑭ $\frac{6}{16}$
⑮ $\frac{4}{10} \times \frac{3}{4}$
⑯ $\frac{6}{9} \times \frac{2}{3}$
⑰ $\frac{12 \cdot 2}{32 \cdot 3}$
⑱ $\frac{30}{80}$
⑲ $\frac{14 \cdot 6}{9 \cdot 9}$
⑳ $\frac{14}{30}$

Fraction - Addition and Subtraction

① $1\frac{1}{24}$
② $\frac{23}{30}$
③ $1\frac{5}{24}$
④ $2\frac{1}{8}$
⑤ $2\frac{4}{5}$
⑥ $7\frac{1}{2}$
⑦ $6\frac{1}{8}$
⑧ $12\frac{1}{3}$
⑨ $5\frac{1}{24}$
⑩ $15\frac{11}{16}$
⑪ $5\frac{7}{9}$
⑫ $7\frac{1}{3}$
⑬ $6\frac{5}{9}$
⑭ $2\frac{37}{40}$
⑮ $\frac{23}{28}$

Fractions and Mixed Numerals

① $\frac{17}{20}$
② $31\frac{5}{12}$
③ $3\frac{1}{3}$
④ $3\frac{7}{24}$
⑤ $4\frac{3}{20}$
⑥ $\frac{4}{9}$
⑦ $\frac{10}{13}$
⑧ $27\frac{5}{8}$
⑨ $1\frac{2}{3}$
⑩ 3
⑪ $\frac{17}{112}$
⑫ $\frac{2}{7}$
⑬ $\frac{23}{38}$
⑭ $\frac{25}{132}$
⑮ $7\frac{5}{48}$
⑯ $3\frac{17}{20}$
⑰ $\frac{4}{27}$
⑱ $\frac{6}{77}$
⑲ ab
⑳ 1
㉑ $\frac{a}{3c}$
㉒ $\frac{a^2b^2}{12}$
㉓ 1

Challenge $\frac{35}{513}$

Mathematics Objective

18 Read and write decimals
19 Compare and order a set of decimals

Teaching Techniques and Introductory Activities

Place value, reading decimal numbers

For teaching suggestions see page 88 - 89 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Comparing decimals

For teaching suggestions see page 90 - 91 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Place value, reading decimal numbers, expanded notation

For teaching suggestions see page 132 - 136 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

note: Teacher's corner p. 135

Comparing decimals

For teaching suggestions see page 137 - 138 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Supplementary Activities and Resources

Place value, expanded numeration

For teaching suggestions see page 70 - 71 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

8 practice p. 28	7 practice p. 37 - 38
8 L - 17	7 - L - 17

Comparing decimals

For teaching suggestions see page 74 - 75 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

8 practice 31	7 practice p. 39
8 L - 20	7 L - 18

For teaching suggestions see page 100 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

p. 111 (1 - 5)
Skillmaster 7 - 26
Skillmaster 8 - 27, 28, 29

Comparing decimals

For teaching suggestions see page 104 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

In Guide: Writing and comparing decimals (T74)

MRC:
MS E 28, E 29, E 36, E 37
AWA II 32, 34
RWAP.

Writing and Comparing Decimals

name ______________________________

For each exercise, draw a ring around the largest number.

1. .85 .098 1.01 .2743
2. 7.72 7.2 7.072 7.277
3. .683 .86 .638 .386
4. 2.005 2.05 2.5 .205
5. .1356 1.536 1.635 1.653
6. 1092.1 1092.01 1092 109.2
7. 94 9.499 90.49 94.9
8. .32 .3 .032 .0032
9. 5.81 5.083 5.8 5.185
10. 4.09 4 4.009 4.4
11. .04712 .74 .40 .107
12. 6.77 6.777 6.7 6.7777

Compare the numbers. Use >, <, or =.

13. .56 _____ .056
14. .007 _____ .07
15. 614.5 _____ 61.45
16. .003 _____ .0030
17. 3.44 _____ 3.4
18. .0084 _____ .084
19. .901 _____ .911
20. 22.9 _____ 22.98
21. 18.65 _____ 18.6
22. 88.2 _____ 88.22
23. .005 _____ .0005
24. .341 _____ .3411
25. 42.5 _____ 42.5008
26. .9573 _____ .9375
27. 67.75 _____ 67.7
28. .8 _____ .800
29. 7.526 _____ 7.52
30. .04033 _____ .04053

Complete each exercise.

31. 7 ones 13 tenths 92 thousandths _8_ and _392 thousandths_ _8.392_
32. 8 ones 12 tenths 5 hundredths _____ and _____________ _____
33. 4 tens 8 ones 7 tenths 14 hundredths _____ and _____________ _____
34. 2 tens 16 tenths 5 hundredths _____ and _____________ _____
35. 1 one 4 hundredths 16 thousandths _____ and _____________ _____

Mathematics Around Us 7, Maintenance page 2. This page may be reproduced.

Multiplying and Dividing Whole Numbers

name ______________________________

Find each missing number.

1. 91 X 25 = __________

2. 45 X 60 = __________

3. 38 X 83 = __________

4. 412 X 900 = __________

5. 651 X 37 = __________

6. 7745 ÷ 5 = __________

7. 4288 ÷ 32 = __________

8. 480 ÷ 25 = __________

9. 541 ÷ 6 = __________

10. 9132 ÷ 761 = __________

11. 5036 X 42 = __________

12. ________ X 45 = 945

13. 108 X ________ = 9396

14. 312 X 273 = __________

15. 194 X 549 = __________

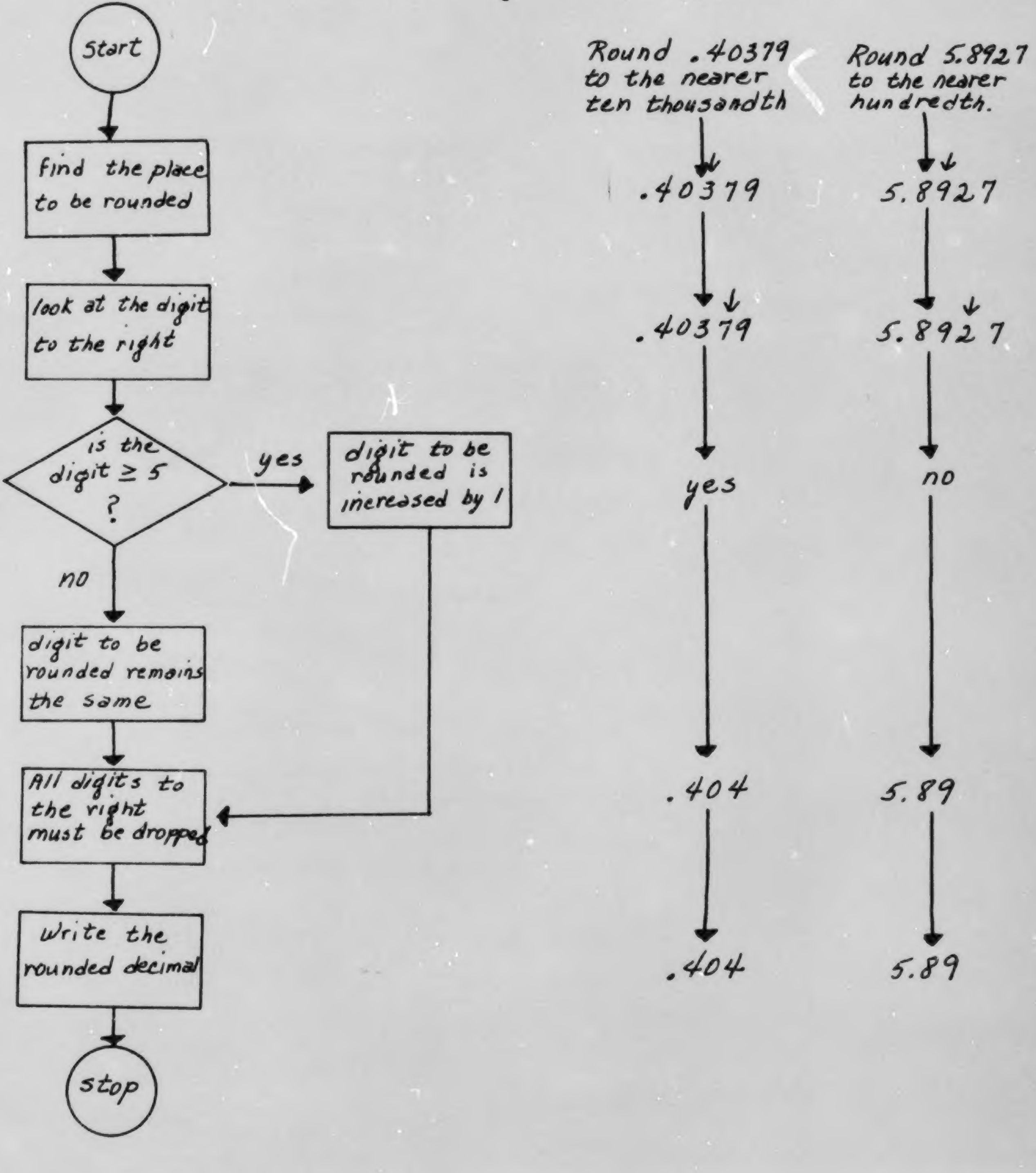
Use anytime after pag T73

Mathematics Objective

20. Round Decimals

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Rounding decimals For teaching suggestions see page 115 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman	Rounding decimals For teaching suggestions see page 76,98 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) 8 practice 46 7 L 27, 29
Rounding decimals For teaching suggestions see page 139 - 140 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	Rounding decimals For teaching suggestions see page 105, 120 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett) Skillmaster 8 - 33 In Guide: To round decimals through ten - Thousandths (a flow - Chart for student notebook) Decimal numerals (Student worksheets with ans)

To round decimals through ten thousandths


Start
find the place to be rounded
look at the digit to the right
is the digit ≥ 5 ?
yes
digit to be rounded is increased by 1
no
digit to be rounded remains the same
All digits to the right must be dropped
Write the rounded decimal
stop
Round .40379 to the nearer ten thousandth
.40379
.40379
yes
.404
.404
Round 5.8927 to the nearer hundredth.
5.8927
5.8927
no
5.89
5.89

Decimal Numerals

Write the following in expanded notation using exponents:

① 2.04 ________________
② 5,709 ________________
③ 147.236 ________________
④ 1,700,069 ________________
⑤ .7004 ________________
⑥ 374.142 ________________

Write the following in standard form

⑦ $7\cdot 10^4 + 6\cdot 10^2 + 2\cdot \frac{1}{10} + 6\cdot \frac{1}{10^2}$ ________________

⑧ $4\cdot 1 + 0\cdot \frac{1}{10} + 5\cdot \frac{1}{10^2} + 3\cdot \frac{1}{10^3}$ ________________

⑨ $5\cdot \frac{1}{10^4} + 7\cdot \frac{1}{10^6}$ ________________

Write the following as decimal fractions

⑩ four thousand, three hundred seventeen and two tenths ________________

⑪ Five and two hundred thirty thousandths ________________

⑫ four thousand six ten-thousandths ________________

⑬ four thousand and six ten-thousandths ________________

⑭ forty-seven millionths ________________

⑮ $\frac{2345}{100} =$ ________________

⑯ $\frac{48}{10000} =$ ________________

⑰ $\frac{23876483}{10000} =$ ________________

⑱ $23\frac{45}{100} =$ ________________

Write the following in words

⑲ .46

⑳ 103.103

㉑ 1200.005

㉒ 1503.6

㉓ 6.23

㉔ 5406

㉕ 50.003

Round to the nearest unit:

㉖ .7   2.01   31.67   .168

Round to the nearest hundred

㉗ 7463   315.04   1,070   917

Round to the nearest hundredth

㉘ 4.607   76.813   .1493   .145

Round to the nearest ten thousandth

㉙ 7.00697   .450632   1.43268   5.67899

Answers

Decimal Numerals

① $2\cdot10^0 + 4\cdot\frac{1}{10}$ or $2\cdot10^0 + 4\cdot10^{-2}$

② $5\cdot10^3 + 7\cdot10^2 + 9\cdot10^0$

③ $1\cdot10^2 + 4\cdot10^1 + 7\cdot10^0 + 2\cdot10^{-1} + 3\cdot10^{-2} + 6\cdot10^{-3}$

④ $1\cdot10^6 + 7\cdot10^5 + 6\cdot10^1 + 9\cdot10^0$

⑤ $7\cdot10^{-1} + 4\cdot10^{-4}$

⑥ $3\cdot10^2 + 7\cdot10^1 + 4\cdot10^0 + 1\cdot10^{-1} + 5\cdot10^{-2} + 2\cdot10^{-3}$

⑦ 706.26
⑧ 4.053
⑨ .000507

⑩ 4317.2
⑪ 5.230
⑫ .4006
⑬ 4000.0006
⑭ .000047
⑮ 23.45
⑯ .0048
⑰ 2387.6483
⑱ 23.45

⑲ forty six hundredths
⑳ one hundred three and one hundred three thousandths
㉑ one thousand two hundred and five thousandths
㉒ one thousand five hundred three and six tenths
㉓ six and twenty-three hundredths.
㉔ five thousand four hundred six
㉕ fifty and three thousandths.

㉖	1	2	32	0
㉗	7500	300	1100	900
㉘	4.61	76.81	.15	.15
㉙	7.0070	.4506	1.4327	5.6790

Mathematics Objective

21M Change a fraction to a terminating or repeating decimal *and conversely*

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
changing fractions to decimals	
p. 250 7 (Scott-Foresman)	p. 138 - 139 8 (Scott-Foresman)
Decimal equivalent for a fractioh	termination or repeating decimals
p. 195 - 197 7 (Silver-Burdett)	p. 242 - 244 8 (Silver-Burdett)
	To express a repeating decimal as a fraction.
	p. 245 - 6 8 (Silver-Burdett)

Express a Repeating Decimal as a Fraction

$.\overline{36}$

$n = .\overline{36}$

Multiply by the power of 10 which will move the decimal point behind a complete pattern. (both sides) $100\,n = 36.\overline{36}$

Multiply both sides by 1. $1\,n = .\overline{36}$

Subtract the last equation from the previous one.

$$\begin{array}{r} 100\,n = 36.36 \\ -\ \ 1\,n = .36 \\ \hline 99\,n = 36 \end{array}$$

Divide both sides by 99. $n = \frac{36}{99}$

Reduce to lowest terms. $n = \frac{4}{11}$

therefore: $.\overline{36} = \frac{4}{11}$

Change the following from decimals to fractions.

① $.\overline{27}$ ② $.\overline{6}$ ③ $.\overline{39}$ ④ $.\overline{312}$

$$\begin{array}{r} 100\,n = 27.\overline{27} \\ 1\,n = .\overline{27} \\ \hline 99\,n = 27 \end{array}$$

$n = \frac{27}{99} = \frac{3}{11}$

Fraction - Decimal Equivalents

① $\frac{3}{7}$ = ______________________ (decimal)

② $\frac{3}{8}$ = ______________________ (decimal)

③ $.\overline{45}$ = ______________________ (fraction)

④ $.\overline{3}$ = ______________________ (fraction)

⑤ $.625$ = ______________________ (fraction)

⑥ $\frac{7}{12}$ = ______________________ (decimals)

⑦ $.\overline{148}$ = ______________________ (fraction)

⑧ $\frac{13}{15}$ = ______________________ (decimal)

⑨ $\frac{1}{3}$ = ______________________ (decimal)

⑩ $.54$ = ______________________ (fraction

Answers

Repeating decimal as a fraction

① $\frac{3}{11}$
② $\frac{2}{3}$
③ $\frac{13}{33}$
④ $\frac{104}{333}$

Fraction - Decimal Equivalents

① $.\overline{428571}$
② $.375$
③ $\frac{9}{11}$
④ $\frac{1}{3}$
⑤ $\frac{5}{8}$
⑥ $.58\overline{3}$
⑦ $\frac{148}{999}$
⑧ $.8\overline{6}$
⑨ $.\overline{3}$
⑩ $\frac{27}{50}$

Mathematics Objective

21M add and subtract decimals

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
adding and subtracting decimals For teaching suggestions see page 92, 94 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)	adding and subtracting decimals For teaching suggestions see page 86 - 87 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) 8 practice p. 37,38 7 practice p. 39 8 - L - 25 7 L - 18
adding and subtracting decimals For teaching suggestions see page 141 - 145 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	adding and subtracting decimals For teaching suggestions see page 106 - 110 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) In Guide: adding and subtracting decimals T 75 MRC: MS: E10, E16, E17, E19 AWA (II) p. 8 - 9, p. 18 - 19, p. 20 - 21

T76 Use anytime after page 97.

Multiplying and Dividing Decimals

name ______________________________

Multiply or divide.

1. 4.6 X 100 = ____________
2. .21 X 1000 = ____________
3. 100 X .451 = ____________
4. .90 X 100 = ____________
5. 1458 X 100 = ____________
6. .5 X 10000 = ____________
7. 732.4 X 1000 = ____________
8. 29.24 X 1000 = ____________
9. 100 X .7891 = ____________
10. 2.222 X 10 = ____________
11. 63 ÷ .09 = ____________
12. 2.5 ÷ .05 = ____________
13. .36 ÷ .006 = ____________
14. .0012 ÷ .003 = ____________
15. .8 ÷ .002 = ____________
16. 4200 ÷ .6 = ____________
17. 5.4 ÷ .09 = ____________
18. 160 ÷ .08 = ____________
19. 3.2 ÷ .004 = ____________

20. 21.83 X .9

21. .18 X .56

22. .213 X .094

23. .35 X .943

24. 32.43 ÷ 6.9

25. .0144 ÷ .12

26. 76.22 ÷ 2.06

27. .01568 ÷ .028

Mathematics Around Us 8, Maintenance page 5. This page may be reproduced.

Adding and Subtracting Decimals

name ________________________________

Add or subtract.

1. 13.21 + 16.8
2. 49 – .731
3. 97.23 + 14.881
4. 6.942 + 283.1
5. 1.001 – .76
6. 2.66 – .6904
7. 2.014 – .967
8. 28.5 – 14.62
9. 8.0004 – 7.35
10. .461 + 5.8 + .0093
11. In the magic square at the right, the sum of the numbers in each row, column, and diagonal is to be 18. Give the missing numbers.

5.78		3.37
	6	

Use anytime after page 87.

Mathematics Objective

21 find the product and quotient of two decimal numbers

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Multiplying decimal numbers For teaching suggestions see page 104,106 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)	For teaching suggestions see page 92 - 94 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) 7 practice p. 43 8 practice 40, 41, 42 7 - H 12 8 L - 27
Dividing decimal numbers For teaching suggestions see page 110,112 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)	Dividing decimals For teaching suggestions see page 96 - 97 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)
Multiplying decimal numbers For teaching suggestions see page 182, 186 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	7 practice 49, 50 7 L 25, 26 Multiplying decimal numbers For teaching suggestions see page 116 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) Skillmaster 7 - 38 Skillmaster 8 - 30

Mathematics Objective

continued:

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Dividing decimal numbers	Dividing decimal numbers
For teaching suggestions see page 192 - 3 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	For teaching suggestions see page 120 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)
	Skillmaster 7 - 43 Skillmaster 8 - 32 Challenge p. 121
	In Guide: Decimals - (Maintaining Skills) Maintaining Skills Multiplying and dividing decimals T 75, T 77

Decimals
Maintaining Skills

① $64.3 + .483 + 7 + 149.76 =$

② $6.8409 \div .09 =$

③ $109.7 - 84.8673 =$

④ $7.034 \times .36 =$

⑤ $48.6321 + 476.04 + 6 + 1.9 =$

⑥ $9003.431 - 934 =$

⑦ $567 \times .086 =$

⑧ $9.041 \times .0256 =$

⑨ $6 \times 7 \times \frac{1}{10^4} =$

⑩ $.00042 \times 100 =$

⑪ $7.004 \times 10^4 =$

⑫ $.06 \times 10^2 =$

⑬ $.093 \times 10^{-3} =$

⑭ $.0032296 \div .008 =$

⑮ $63.0493 \div 5.63 =$
(nearest hundredth)

⑯ $.70419 \div .27 =$
(nearest hundredth)

Maintaining Skills

① $5\frac{1}{4} \div 2\frac{1}{3} =$

② $\frac{16}{36} \cdot \frac{15}{24} \cdot \frac{9}{10} \cdot \frac{4}{5} =$

③ $3\frac{1}{10} + 2\frac{1}{2} + 4\frac{3}{4} + 5\frac{1}{5} =$

④ $1\frac{2}{3} \cdot 8\frac{2}{3} \cdot \frac{1}{5} =$

⑤ $.0073 =$ ____________ (fraction)

⑥ $\frac{789}{100000} =$ ____________ (decimal)

⑦ $3.94 =$ ____________ (fraction)

⑧ $7.714 + .042 + 48 + 1.6 =$

⑨ $1613.2 - 798.0493 =$

⑩ $96.473 - 84.89 =$

⑪ $21 - 7\frac{7}{9} =$

⑫ $2\frac{3}{4} \cdot 5\frac{1}{3} \cdot \frac{9}{10} =$

⑬ $4\frac{5}{6} \div 3\frac{1}{3} =$

⑭ $5\frac{2}{3} + 7\frac{2}{5} + 4\frac{5}{6} =$

⑮ $6 \cdot 10^{2} + 3 \cdot 10^{1} + 2 \cdot 10^{0} + 9 \cdot 10^{-1} + 3 \cdot 10^{-2} + 4 \cdot 10^{-3} =$

⑯ Round 934.05487 to the nearest

unit ________ hundred ________ hundredth ________ ten-thousandth ________

⑰ $3 \cdot 2^{3} \div 6 \cdot 5 \div 2^{2} \cdot 16 =$

⑱ $\frac{36 \div 9 + 4 \times 5}{3(8 + 10)} =$

Answers

Decimals (Maintaining skills)

① 221.543
② 76.01
③ 24.8327
④ 2.53224
⑤ 532.5721
⑥ 8069.431
⑦ 48.762
⑧ .2314496
⑨ .0042
⑩ .042
⑪ 70040
⑫ 6
⑬ .000093
⑭ .4037
⑮ 11.20
⑯ 2.61

Maintaining skills

① $2\frac{1}{4}$
② $\frac{1}{5}$
③ $15\frac{11}{20}$
④ $2\frac{8}{9}$
⑤ $\frac{73}{10000}$
⑥ .00789
⑦ $\frac{197}{30}$
⑧ 57.356
⑨ 815.1507
⑩ 11.583
⑪ $13\frac{2}{9}$
⑫ $13\frac{1}{5}$
⑬ $1\frac{9}{20}$
⑭ $17\frac{9}{10}$
⑮ 632.934
⑯ 934 900 934.05

934.0549

⑰ 80
⑱ $\frac{24}{57} = \frac{8}{19}$

T78 Use anytime after page 235.

Adding and Subtracting Mixed Numbers

name ______________________

Add.

1. $1\frac{5}{6}$
$+ 2\frac{5}{12}$

2. $2\frac{1}{4}$
$+ 3\frac{2}{3}$

3. $2\frac{1}{5}$
$+ 1\frac{7}{10}$

4. $4\frac{5}{6}$
$+ 5\frac{3}{4}$

5. $3\frac{5}{6}$
$+ 1\frac{4}{9}$

6. $14\frac{1}{4}$
$+ 10\frac{2}{5}$

Subtract.

7. $4\frac{4}{9}$
$- 3\frac{1}{3}$

8. $5\frac{2}{3}$
$- 2\frac{1}{6}$

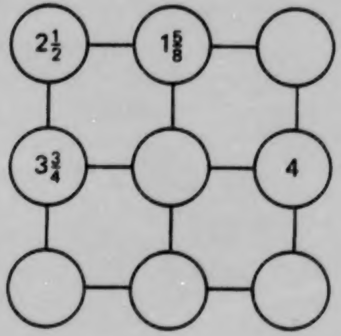
9. $5\frac{1}{2}$
$- 2\frac{2}{3}$

10. 7
$- 3\frac{3}{8}$

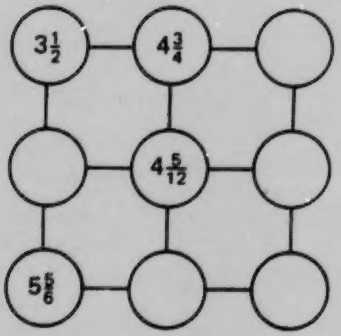
11. $4\frac{1}{4}$
$- \frac{2}{5}$

12. $28\frac{2}{3}$
$- 15\frac{4}{5}$

13. The sum of the numbers in each row and in each column is to be $8\frac{3}{4}$. Give the missing numbers.



14. The sum of the numbers in each row and in each column is to be $10\frac{5}{12}$. Give the missing numbers.



Mathematics Around Us 7, Maintenance page 6. This page may be reproduced.

Dividing Decimals

name ____________________________

Write each exercise with a whole number divisor. Then find the quotient.

1. 20 ÷ .05

________ ÷ 5 = ________

2. .56 ÷ .7

________ ÷ 7 = ________

3. 4 ÷ .005

________ ÷ 5 = ________

4. .497 ÷ .14

________ ÷ 14 = ________

5. .684 ÷ 1.2

________ ÷ 12 = ________

6. 36.1 ÷ .95

________ ÷ 95 = ________

7. 24 ÷ .096

________ ÷ 96 = ________

8. 6 ÷ .0004

________ ÷ 4 = ________

Divide.

9. $39\overline{)285.09}$

10. $7\overline{).994}$

11. $5.2\overline{)47.112}$

12. $.0032\overline{).644}$

13. $2.435\overline{)3.1655}$

14. $52.7\overline{)237.15}$

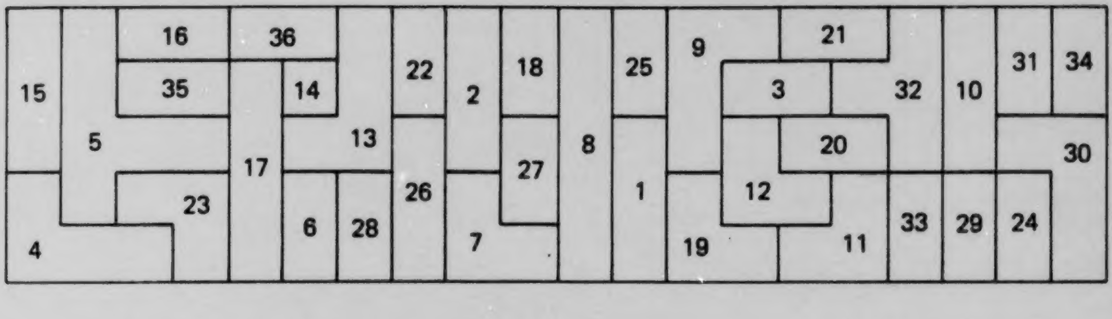
15. $8.4\overline{)1.596}$

16. $13.8\overline{).0966}$

Mathematics Around Us 8, Maintenance page 2. This page may be reproduced.

Computing with Integers

name ______________________________


16
36
21
15
35
14
22
2
18
25
9
3
32
10
31
34
5
13
20
30
17
8
27
26
1
12
23
6
28
7
19
11
33
29
24
4

For each exercise compute the answer. If the answer is greater than $^{-}2$, shade the region above that corresponds to the exericse.

1. $^{-}6 + ^{-}4 =$ ________

2. $7 - ^{-}4 =$ ________

3. $24 \div ^{-}6 =$ ________

4. $^{-}5 + 5 =$ ________

5. $^{-}15 + ^{-}4 =$ ________

6. $^{-}10 + 1 =$ ________

7. $11 - ^{-}3 =$ ________

8. $1 \times ^{-}1 =$ ________

9. $^{-}20 \times ^{-}3 =$ ________

10. $^{-}12 \div ^{-}4 =$ ________

11. $^{-}13 - ^{-}14 =$ ________

12. $^{-}16 + ^{-}3 =$ ________

13. $^{-}4 \times ^{-}7 =$ ________

14. $^{-}6 \times 2 =$ ________

15. $^{-}11 \times ^{-}3 =$ ________

16. $^{-}45 \div 9 =$ ________

17. $^{-}75 - ^{-}75 =$ ________

18. $^{-}15 \times 6 =$ ________

19. $10 \div ^{-}10 =$ ________

20. $^{-}5 \times ^{-}4 =$ ________

21. $30 - ^{-}9 =$ ________

22. $^{-}16 + 1 =$ ________

23. $^{-}9 \times 2 =$ ________

24. $^{-}4 - 29 =$ ________

25. $2 \times ^{-}55 =$ ________

26. $0 - 106 =$ ________

27. $60 \div ^{-}30 =$ ________

28. $60 - ^{-}8 =$ ________

29. $^{-}39 \div ^{-}3 =$ ________

30. $^{-}45 + 44 =$ ________

31. $40 \div ^{-}10 =$ ________

32. $125 \div ^{-}5 =$ ________

33. $^{-}3 \times 60 =$ ________

34. $150 - ^{-}43 =$ ________

35. $^{-}300 + ^{-}150 =$ ________

36. $^{-}84 \div ^{-}12 =$ ________

T74 Use anytime after page 79.

Multiplying by Powers of Ten

name ____________________

For this cross-number puzzle you are to decide where the answer goes in the puzzle. Shade in squares that are not used. Do not put decimal points in the puzzle.

1 3	2 2	3 1	4 0	0	■	5 1	6 2	7 5	8 1	9 6	■
10 4					11						12
13 0				14							
15 0			16		17						
■	■	18		19		20				21	
22	23		24		25					26	27
28		29		30			31	32			
33				34		35					
36			37		38	39				40	
	41			42				43			
		44				45					

ACROSS

1. 32.1×10^3
5. 125.16×10^{-3}
10. $.4121 \times 10^4$
11. 8.13×10^6
13. $.3 \times 10^{-2}$
14. 349.128×10^{-5}
15. 15.9×10^{-3}
17. $.2005 \times 10^{-1}$
18. 21×10^{-3}
20. $.76 \times 10^5$
22. 9.3×10
24. 5.3×10^4
26. 3000×10^{-2}
28. $.243 \times 10^3$
30. 2800×10^{-1}
31. $.184 \times 10^4$
33. 2×10^{-3}
34. 4×10^{-2}
35. 32.25×10^3
36. 56×10^{-4}
38. $.3 \times 10^{-3}$
40. $.8 \times 10^2$
41. $.3129 \times 10^{-3}$
43. 3×10^2
44. $.28 \times 10^{-2}$
45. 5.1042×10^4

DOWN

1. $.34 \times 10^4$
2. $.2101 \times 10^4$
3. 123.5×10^2
4. 1×10^{-2}
5. $11{,}327 \times 10^2$
6. 23.406×10^4
7. 5.09×10^4
8. $.1015 \times 10^5$
9. 602×10^{-3}
11. 8×10^2
12. 8×10^{-2}
16. 9.25×10^2
19. $.132 \times 10^4$
21. 3.45804×10^{-2}
22. 92×10^2
23. 34×10^3
25. $84{,}018 \times 10^{-6}$
27. 2×10^{-6}
29. 3.25×10^4
31. 1230×10^{-1}
32. 8200×10^{-2}
35. $.3095 \times 10^4$
37. $.6 \times 10^3$
39. 2×10^{-2}
42. $.32 \times 10^2$
43. $.03 \times 10^3$

Mathematics Around Us 8, Maintenance page 3. This page may be reproduced.

Mathematics Objective

22 Multiply and divide by powers of ten

21M Change fractions to terminating and repeating decimals and conversely.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Multiply and divide by powers of ten For teaching suggestions see page 108,118 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)	For teaching suggestions see page 72 - 73 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) (negative exponents) 7 practice p. 46, 47, 53, 54 7 - L 30 7 - L 23
Changing fractions to decimals For teaching suggestions see page 250 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)	Changing fractions to decimals For teaching suggestions see page 137 - 139 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) 7 practice p. 92 8 practice p. 62 7 - H 25 8 - H 15
Multiply and divide by powers of ten p. 188, 190 7 (Silver Burdett) p. 102, 260 8 (Silver Burdett)	8 - H 15 is an exercise for expressing repeating decimals as fractions. In Guide: Multiplying by powers of ten T 74 MRC: The math wizard 4 p. 52 - 3 puzzers AWA - 36 - 38 MS - E 38, E 39 PWAP - AA 51

Mathematics Objective

23. Use a ratio to compare two quantities

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Equal ratios For teaching suggestions see page 260-61 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman) Equal ratios For teaching suggestions see page 244-248 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	For teaching suggestions see page 260 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) 7 practice p. 96 8 practice p. 95 For teaching suggestions see page 154-156 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) Skillmaster 8-37 In Guide: Comparing Measures MRC: AWA p. 40, 42

Comparing Measures

Name each ratio as a fraction in simplest form:

1) 8 oz. to 3 lbs.
2) 82 cm to 3 m
3) 15 min. to 1 hr.
4) 6 yd. to 48 ft.
5) 50 min. to 2 days
6) 8 qt. to 4 gal.
7) 6 mo. to 2 yr.
8) 25¢ to $1.50
9) 1000 lb. to 4 T
10) 5'2" to 4'10"
11) 4'3" to 2'9"
12) $3\frac{1}{2}$ ft. to $1\frac{1}{2}$ yd.
13) 18 in. to 1 yd.
14) 1320 ft. to 2 mi.
15) 4 oz. to 3 lb.
16) 1 week to 1 yr.

United States System

16 fluid ounces (fl. oz.) make 1 pint (pt.)
2 pints make 1 quart (qt.)
4 quarts make 1 gallon (gal.)

United States System

16 ounces (oz.) make 1 pound (lb.)
2000 pounds make 1 ton (T.)
2240 pounds make 1 long ton

United States System

12 inches (in.) make 1 foot (ft.)
36 inches, or 3 feet, make 1 yard (yd.)
$16\frac{1}{2}$ feet make 1 rod (rd.)
5280 feet, or 1760 yards, make 1 mile (mi.)

Conparing Measures

(Key)

1) $\frac{1}{6}$
2) $\frac{41}{150}$
3) $\frac{1}{4}$
4) $\frac{3}{8}$
5) $\frac{5}{288}$
6) $\frac{1}{4}$
7) $\frac{1}{2}$
8) $\frac{1}{6}$
9) $\frac{1}{8}$
10) $\frac{31}{29}$
11) $\frac{17}{11}$
12) $\frac{7}{9}$
13) $\frac{1}{2}$
14) $\frac{1}{8}$
15) $\frac{1}{12}$
16) $\frac{1}{52}$

Mathematics Objective

24. Solve a proportion
25. Use a proportion to solve a problem

Teaching Techniques and Introductory Activities

Using Proportions
For teaching suggestions see page 264-273 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)

Solving Proportions
For teaching suggestions see page 249-53 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)

Using Proportions
For teaching suggestions see page 244-259 of the Teacher's Edition of *Mathematics for Mastery* Silver Burdett

Supplementary Activities and Resources

Using Proportions
For teaching suggestions see page 260-71 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

7 practice 95, 96, 97, 98 7-H 27, H 28
8 practice 96, 97 8-L-51

For teaching suggestions see page 157-159 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

Skillmaster 8-38

Using Proportions
For teaching suggestions see page 154-167 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)

Skillmaster 8-39, 40

In Guide: Ratio and Proportion
Using Proportions: Map T79

MRC:
Amusements in Developing Geometry Skills
Vol. III p. 27

PAWPS: BB 29 BB 32
AA 34
BB 30
BB 31

Ratio and Proportion

Complete each proportion:

① $\frac{42}{\quad} = \frac{28}{30}$　② $\frac{24}{63} = \frac{16}{\quad}$　③ $\frac{\quad}{21} = \frac{10}{35}$

④ $\frac{5}{6} = \frac{n}{n+4}$, $n =$　⑤ $1 : \frac{2}{3} = 1\frac{1}{2} : n$　$n =$　⑥ $1\frac{1}{2} : 2 = n : 3$　$n =$

⑦ $1 : 1\frac{1}{2} = n : 1\frac{7}{8}$　$n =$　⑧ $1\frac{1}{4} : 1\frac{1}{2} = y : 1\frac{1}{8}$　$y =$

Solve the following:

⑨ On a map, 3 inches represents 50 miles. How many miles are represented by $19\frac{1}{2}$ inches?

⑩ At Harry Wilson Middle School the ratio of boys to girls is 5 to 4. If there are 760 boys, how many girls are there?

⑪ This week oranges are sale priced at 5 pounds for $2.50. How much will 13 pounds of oranges cost?

⑫ Mr. Brown can drive 260 miles on 15 gallons of gasoline. How far can he drive on 22 gallons?

⑬ At the basketball game, the ratio of boys to girls was 3 to 1. If 174 boys attended, how many girls attended?

⑭ At a career conference held recently, the ratio of those interested in law to those interested in medicine was 5 to 3. If 760 people were interested in law, how many were interested in medicine?

⑮ Which is the better buy, 16 for $19.68 or 3 for $3.72?

⑯ Which is the better buy, 4 for $3.40 or 6 for $4.95?

⑰ If an item is priced at 5 for $3.25, how much would 7 cost?

(18) A recipe for making 12 pancakes calls for $2\frac{1}{4}$ cups of flour. How much flour is needed to make 1 pancakes?

⑲ A man is paid $76 for 8 hours of work. How much should this man be paid for working 37 hours at the same hourly wage?

⑳ Tom rode 12 miles on his bicycle during the same time Pete rode 9 miles. How far did Pete ride during the time Tom rode 4 miles?

21 The earth moves in its orbit around the sun at the rate of $18\frac{1}{2}$ miles per second. What is the rate in miles per hour?

Answers

Ratio and Proportion:

① 45 ② 42 ③ 6

④ 20 ⑤ 1 ⑥ $2\frac{1}{4}$

⑦ $1\frac{1}{4}$ ⑧ $1\frac{9}{16}$

⑨ 325 mi.
⑩ 608 girls
⑪ \$6.5
⑫ $381\frac{1}{3}$ miles
⑬ 58 girls
⑭ 456 people
⑮ 16 for \$19.68
⑯ 4 for \$3.40
⑰ \$4.55
⑱ $3\frac{3}{8}$
⑲ \$351.50
⑳ 3 mi
㉑ 66,600 mph

T80 Use anytime after page 293.

Percent

name ____________________

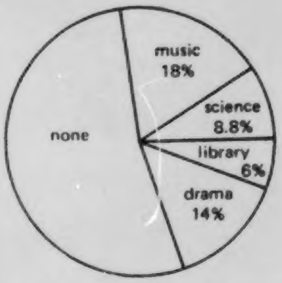
1. The clubs shown in the graph meet at the same time, so a student can join only one club. The total enrollment is 1250. Find the number in each club.

Drama: ________

Music: ________

Science: ________

Library: ________

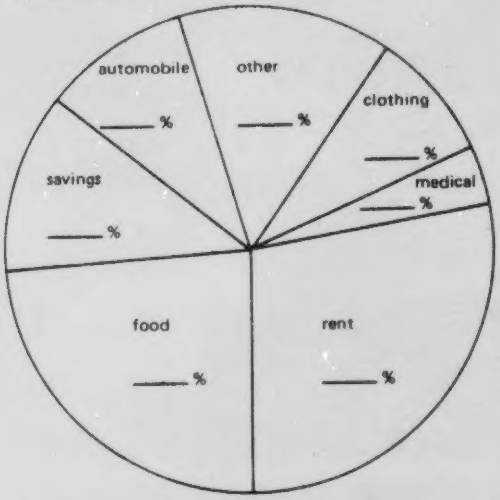


2. Give the percent and the number of students in none of the four clubs.

________ %   ________ students

3. Mr. Adkin's monthly take-home pay is $950. The information shows how he used the money. Put the percents in the graph.

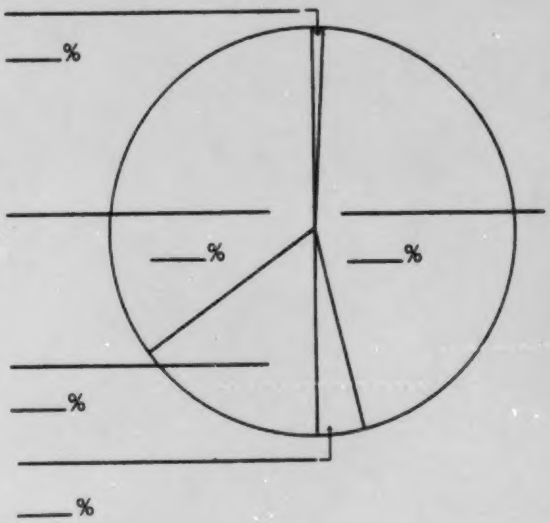
Rent: $266
Food: $228
Clothing: $76
Automobile: $95
Savings: $114
Medical: $38
Other: $133



4. The 500 students at LaSalle Junior High School support football in the percents shown below. A student can be in only one of the groups.

Football team: 4%
Band: 15%
Camera crew: 1%
Pep club: 35%
None: 45%

Label each section of the circle graph with the name of a group and a percent.



5. Give the number of students in each group.

Football team: ________

Band: ________

Camera crew: ________

Pep club: ________

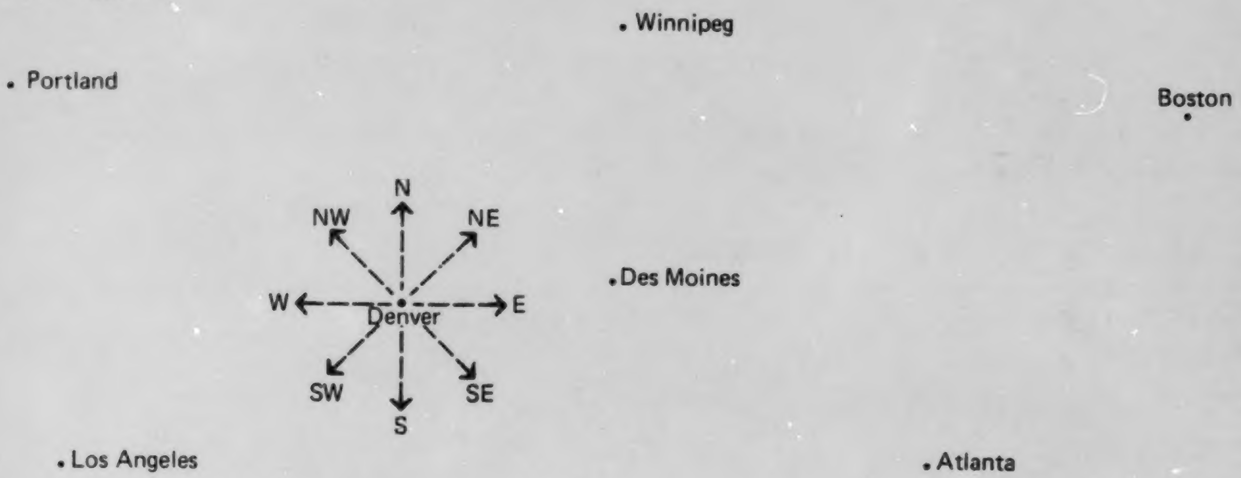
None: ________

Mathematics Around Us 8, Maintenance page 9. This page may be reproduced.

Mathematics Around Us 8, Maintenance page 8. This page may be reproduced.

Using Proportions: Maps name ______________________

The map shows approximate air distances from Denver.
1 cm = 225 km



For each city, find the distance on the map to the nearest tenth of a centimeter. Then find the actual distance to the nearest kilometer. Round .5 km up.

Denver to:	Map Distance	Actual Distance
1. Los Angeles	________ cm	________ km
2. Atlanta	________ cm	________ km
3. Portland	________ cm	________ km
4. Winnipeg	________ cm	________ km
5. Boston	________ cm	________ km
6. Des Moines	________ cm	________ km

Put each city on the map. Find the map distance from Denver to the nearest tenth of a centimeter. Then use the arrows around Denver to help you locate the city.

Denver to:	Actual Distance	Map Distance
7. Richmond	2400 km E	________ cm
8. Ciudad Juarez	900 km S	________ cm
9. Sacramento	1450 km W	________ cm
10. Duluth	1300 km NE	________ cm
11. Vancouver	1800 km NW	________ cm
12. Cheyenne	150 km N	________ cm

Mathematics Objective

26. Rename fractions, decimals, and percents as equivalent values

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 276-281 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman) For teaching suggestions see page 260, 268 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett)	For teaching suggestions see page 283-285 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) practice 7-99, 100, 101 practice 8-101, 102 8-H29 For teaching suggestions page 169, 173 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) Skillmaster 8, p. 41 In guide: Percents - notes Fraction - Decimal - percent MRC: AWA (III) pp. 36, 38, 40, 44 PWAP BB 34, BB 37, BB 38, BB 39, BB 33

PERCENTS

Consider the following:

$$1 = \frac{100}{100} = 100\% \qquad \frac{56}{100} = 56\% \qquad \frac{24}{100} = 25\%$$

$$1 = \frac{100}{100} = \frac{100}{1} \cdot \frac{1}{100} = 100\% \qquad \frac{56}{100} = \frac{56}{1} \cdot \frac{1}{100} = 56\%$$

The percent symbol (%) replaces the fraction $\frac{1}{100}$ which is the reciprocal of 100. Conversely, when the percent symbol is removed, it is replaced by the fraction bar over a denominator of 100 or times the reciprocal of 100, $\frac{1}{100}$.

Any number can be multiplied by 1 without changing its value, so a number can be multiplied by 100% without changing its value. This gives us another method for renaming a whole number, fraction or a decimal as a percent.

$$.06 \times 1 = .06 \times 100\% = 6\%$$

$$\frac{2}{7} \times 1 = \frac{2}{7} \times 100\% = \frac{200}{7}\% = 28\frac{4}{7}\%$$

$$5 \times 1 = 5 \times 100\% = 500\%$$

$$\frac{1}{8} \times 1 = \frac{1}{8} \times 100\% = \frac{100}{8}\% = 12\frac{1}{2}\%$$

Changing a percent to a fraction or decimal

$$16\% = \frac{16}{100} = \frac{4}{25} \quad \text{or} \quad 16\% = \frac{16}{100} = .16$$

$$\frac{2}{3}\% = \frac{2}{3} \cdot \frac{1}{100} = \frac{2}{300} = \frac{1}{150} \quad \text{or} \quad \frac{2}{3}\% = \frac{2}{3} \cdot \frac{1}{100} = \frac{2}{300} = .006\overline{6}$$

$$8\frac{1}{3}\% = 8\frac{1}{3} \cdot \frac{1}{100} = \frac{25}{3} \cdot \frac{1}{100} = \frac{1}{12} \quad \text{or} \quad 8\frac{1}{3}\% = 8\frac{1}{3} \cdot \frac{1}{100} = \frac{25}{3} \cdot \frac{1}{100} =$$

$$\frac{25}{300} = .08\overline{3}$$

Fraction - Decimal - Percent

Fraction		decimal	Percent
①			$55\frac{1}{2}\%$
②	$\frac{7}{16}$		
③		4.25	
④			$\frac{3}{4}\%$
⑤	$\frac{1}{9}$		
⑥			$16\frac{2}{3}\%$
⑦		.0035	
⑧	$\frac{3}{4}$		
⑨		.175	
⑩	$\frac{4}{7}$		
⑪		.95	
⑫			$\frac{1}{2}\%$

Answers

	Fraction	Decimal	Percent
①	$\frac{111}{200}$	.555	
②		.4375	$43\frac{3}{4}\%$
③	$\frac{17}{4}$		425%
④	$\frac{3}{400}$	.0075	
⑤		$.\overline{1}$	$11\frac{1}{9}\%$
⑥	$\frac{1}{6}$	$.1\overline{6}$	
⑦	$\frac{7}{2000}$		$\frac{7}{20}\%$
⑧		.75	75%
⑨	$\frac{7}{4}$		17.5%
⑩		$.\overline{571428}$	$57\frac{1}{7}\%$
⑪	$\frac{19}{20}$		95%
⑫	$\frac{1}{200}$	.005	

Mathematics Objective

27. Solve equations and word problems which can be reduced to equations of the form p = rb for p, r, or b given any two variable values

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 282, 286 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman)	For teaching suggestions see page 286-290 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)
p. 287 shows the student different ways for finding precent of a number.	8 practice p. 105, 106 8 H30, 31
For teaching suggestions see page 270, 273, 276 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)	For teaching suggestions see page 174-176 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)
	Skillmaster 8, p. 43
	In Guide: Solving percent problems Percents, Decimals, Fractions Percent cross number puzzle # 1 and # 2
	MRC: AWA II p. 22, 24, 26
	PAWP BB 42, BB 46, BB 49 BB 35, BB 45

Solving Percent Problems

Using proportion:

$$\text{percentage} = \text{rate} \times \text{base}$$

$$\frac{\text{percentage}}{\text{base}} = \frac{n}{100} = \text{rate}$$

$$\frac{\text{is}}{\text{of}} = \frac{\text{percentage}}{\text{base}} = \frac{n}{100} = \text{rate}$$

$\text{rate} = \frac{n}{100} = n\%$

percentage compares with base as a number compares with 100.

percentage is a part of the base.

Using equation:

40 is 25% of what number?

$40 = \frac{25}{100} \cdot$ or $40 = .25$

$40 \cdot \frac{100}{25} =$

$160 =$

What percent of 40 is 25?

$n \cdot \frac{1}{100} \cdot 40 = 25$

$n = \frac{25}{1} \cdot \frac{100}{40}$

$n = 62\frac{1}{2}$

$62\frac{1}{2}\%$

40 percent of 25 is what number

$\frac{40}{100} \cdot 25 = p$

$10 = p$

Using proportion:

40 is 25% of what number?

$\frac{\text{is}}{\text{of}} = \frac{n}{100}$ $\frac{40}{b} = \frac{25}{100}$

$40 \cdot 100 = 25 \cdot b$

$\frac{4000}{25} = b$

$160 = b$

What percent of 40 is 25?

$\frac{25}{40} = \frac{n}{100}$

$\frac{25 \cdot 100}{40} = n$

$62\frac{1}{2} = n$

$62\frac{1}{2}\% = n\%$

40 percent of 25 is what number?

$\frac{p}{25} = \frac{40}{100}$

$p = \frac{25 \cdot 40}{100}$

$p = 10$

Percents, Decimals, Fractions

Rename as indicated:

① $\frac{3}{7}$ = __________ (decimal)

② 16% = __________ (fraction)

③ $\frac{7}{8}$ = __________ (percent)

④ $\frac{1}{32}$ = __________ (decimal)

⑤ 1.5 = __________ (percent)

⑥ $\frac{2}{3}$ = __________ (decimal)

⑦ .6% = __________ (decimal)

⑧ 144% = __________ (fraction)

⑨ $\frac{6}{250}$ = __________ (percent)

⑩ 4 = __________ (percent)

⑪ 7.4% = __________ (decimal)

⑫ $4\frac{1}{4}$% = __________ (decimal)

⑬ $\frac{2}{3}$% = __________ (fraction)

⑭ .615 = __________ (fraction)

⑮ $83\frac{1}{3}$% = __________ (fraction)

Solve the following.

⑯ 19% of 72 is what number? __________

⑰ 7 is what percent of 16? __________

⑱ 90 is 120% of what number? __________

⑲ What percent of 80 is 100? __________

⑳ What is 22% of 80? __________

㉑ 31% of what number is $15\frac{1}{2}$? __________

㉒ $5\frac{2}{3}$% of what number is $4\frac{1}{4}$? __________

Percents, Decimals, Fractions

(Key)

① $.\overline{428571}$
② $\frac{4}{25}$
③ $87\frac{1}{2}\%$
④ .03125
⑤ 150%
⑥ $.\overline{6}$
⑦ .006
⑧ $\frac{36}{25}$
⑨ 2.4%
⑩ 400%
⑪ .074
⑫ .0425
⑬ $\frac{1}{150}$
⑭ $\frac{123}{200}$
⑮ $\frac{5}{6}$
⑯ $13\frac{17}{25}$
⑰ $43\frac{3}{4}\%$
⑱ 75
⑲ 125%
⑳ 17.6
㉑ 50
㉒ 75

Mathematics Objective

27. Solve equations and word problems which can be reduced to equations of the form $p = r \cdot b$ for p, r, or b given any two variable values.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Percentage 288-291 7 (Scott Foresman)	p. 285-289, p. 300 8 (Scott Foresman) Practice 8 p. 104
Percentage p. 277-286 7 (Silver Burdett) p. 434 (5, 6, 7, 8)	p. 174-182 8 (Silver Burdett) p. 433 (4-21)
	In Guide: 1. Percentage formulas 2. Percentage problems (ans) 3. Compound interest a La Simple calculator
	MRC: PAWP. BB 42-51

Interest

Mr. Jones is going to invest $1600. At the end of two years this money will be returned to him along with the interest it has earned. He has a choice between two ways of investing the money. In both situations the rate of interest is 12% per year, but the interest is computed in different ways.

In one situation his interest would be computed by determining the product of 1600 (the principal in dollars), the rate of interest (12%), and the time in years (2). Interest computed in this way is called simple interest.

In the other situation his interest on $1600 at 12% would be computed at the end of six months. This interest would be added to the principal. Interest at the end of the first year would be computed on the increased principal. The procedure would be repeated during the second year. Interest computed in this way is called compound interest. Mr. Jones' money would be earning interest at the rate of 12% compounded semiannually (twice a year).

What $1600 will earn in two years:

Simple Interest

Interest = principal x rate x time

$$I = (1600)(.12)(2)$$

$$= 384$$

Amount at the end of two years will be:
principal + interest or $1600 + $384 = $1984

The formula $A=p(1+r)^n$ can be used to find the amount of the principal at the end of a given number of periods. "A" represents the amount; "p" represents the original principal; "r" represents the rate of interest for one period; and "n" represents the given number of periods.

Using the formula we would have computed compound interest on Mr. Jones' investment in this manner:

$$A = p(1+r)^n$$

$$A = 1600\left[1+(.12)(\tfrac{1}{2})\right]^4$$

$$= 1600(1.06)^4$$

$$= 2019.97$$

Amount at the end of two years (4 periods compounded semiannually) will be #2019.97

Compute the interest compounded quarterly on #2000 invested at $7\frac{1}{2}\%$. What will the amount of the principal be at the end of a year and a half?

Compound Interest

Interest compounded semiannually

Interest = principal x rate x time

$I = (1600)(.12)(\frac{1}{2})$

$I = 96$

$p = 1600 + 96$

$= 1696$ (principal at the end of the first six months)

$I = (1696)(.12)(\frac{1}{2})$

$I = 101.76$

$p = 1696 + 101.76$

$= 1797.76$ (principal at the end of the first year)

$I = (1797.76)(.12)(\frac{1}{2})$

$I = 107.87$

$p = 1797.76 + 107.87$

$= 1905.63$ (principal at the end of a year and a half)

$I = (1905.63)(.12)(\frac{1}{2})$

$= 114.34$

$P = 1905.63 + 114.34$

$= 2019.97$ (principal at the end of two years)

Amount at the end of two years will be \$2019.97

Percentage

percentage = rate x base

Interest = principal x rate x time

Discount = original price x rate of discount
Sale price = original price − discount

Commission = rate of commission x sales
Total salary = base salary + commission

Percentage Problems

① The income of the Wilson family is \$1250 per month. They spend \$400 a month or rent and utilities. What percent of their income is spent on these items? ____________

② Find the interest on \$ 2,475 invested at a rate of 6 % for 7 years. ____________

③ Pete earns a base salary of \$1000 and receives a commission at the rate of 4% on his sales. What is his total salary for the month if he sells \$19,500 worth of goods? ____________

④ Sixty-five percent of the students at Abbey Middle School are boys. If there are 1430 boys, what is the total number of students? ____________

⑤ Patti receives a base salary of \$200 a week plus 1% commission on sales. If her total salary for a particular was \$232.75, what were her sales for the week? ____________

⑥ What is the sale price on an item originally priced at \$69.50 that is reduced 40%? ____________

⑦ A desk originally sold for \$175. It is marked down to \$122.50. What was the discount? ______
What was the rate of discount? ____________

⑧ Joe deposited \$600 in a savings account that pays interest at the rate of $4\frac{3}{4}$% compounded semiannually. How much money was there in the account at the end of 18 months? ____________

⑨ Bob Carson borrowed $895 for 1 year. He paid $6\frac{1}{2}$% interest. What amount of interest did he pay?

____

⑩ An automobile tire sells for $48. The profit is $6. What percent of the selling price is the profit?

____

⑪ Mrs. Sands borrowed $3200 to help pay for a new car. The loan was to be repaid in one payment at the end of $1\frac{1}{2}$ years. The yearly rate of interest was $9\frac{1}{2}$%. How much interest must Mrs. Sands pay?

____

⑫ Mr. West sold a house for $42,500. He received a 6% commission. What was the amount of his commission?

____

⑬ John paid $80 interest over a two year period on a certain amount of money he had borrowed. If the interest rate was 8% per year, how much money did he borrow?

____

14 The sale price of a $324 bicycle is $243. What percent of the original price is the bicycle reduced?

____

⑮ A man paid $119 interest on an $850 loan. The rate of interest was 8% per year. For how long was the money borrowed?

____

⑯ Mr. Harvey bought stock at $80 a share. The price rose to $100 a share. What was the percent of increase?

____

⑰ A damaged tennis racket is selling for $21. This is a discount of 30%. What was the original price of the racket?

⑱ Mrs. Bass, a salesperson, receives a 15% commission on her sales. How much must she sell to earn $180?

⑲ How much simple interest will $4000 earn in 5 years and 6 months at $4\frac{1}{2}$%?

⑳ On a vacation Gerald bought a souvenir priced at $16.00. He paid $16.96, which included the state sales tax. How much was the sales tax?
What was the sales tax rate?

㉑ Mrs. Hall has $14,900 taxable income. Her income tax was $2059 plus 26% of her taxable income over $12,400. What was her total income tax?

Extra Problems

① $\frac{4}{25}$ = ______%

② 9 = ______%

③ $\frac{5}{6}$ = ______%

④ 5.6 = ______%

⑤ .0006 = ______%

⑥ $2\frac{3}{5}$ = ______%

	fraction	decimal
⑦ .9% =		
⑧ 325% =		
⑨ $62\frac{1}{2}$% =		
⑩ 32% =		
⑪ $\frac{4}{5}$% =		
⑫ $\frac{3}{8}$% =		
⑬ $\frac{7}{16}$% =		
⑭ .74% =		
⑮ 35% =		
⑯ $3\frac{1}{2}$% =		
⑰ $\frac{5}{9}$% =		
⑱ $\frac{5}{6}$% =		

⑲ 125 is 40% of what number? ______

⑳ 16 is 125% of what number? ______

㉑ 31% of what number is $15\frac{1}{2}$? ______

㉒ 5 is what percent of 16? ______

Answers

Extra problems:

① 16 %
② 900 %
③ $83\frac{1}{3}$ %
④ 560 %
⑤ .06 %
⑥ 260 %
⑦ $\frac{9}{1000}$; .009
⑧ $\frac{13}{4}$, 3.25
⑨ $\frac{5}{8}$, .625
⑩ $\frac{8}{25}$, .32
⑪ $\frac{1}{125}$, .008
⑫ $\frac{3}{800}$, .00375
⑬ $\frac{7}{1600}$, .004375
⑭ $\frac{37}{5000}$, .0074
⑮ $\frac{7}{20}$, .35
⑯ $\frac{7}{200}$, .035
⑰ $\frac{5}{900}$, $.00\overline{5}$
⑱ $\frac{1}{120}$, $.008\overline{3}$
⑲ $312\frac{1}{2}$
⑳ $12\frac{4}{5}$
㉑ 50
㉒ $31\frac{1}{4}$ %

Answers

Percentage Problems

① 32%
② $1037.50
③ $1780
④ 2200
⑤ $3275
⑥ $41.70
⑦ $52.50; 30%
⑧ $643.77
⑨ $58.18
⑩ $12\frac{1}{2}$%
⑪ $456
⑫ $2550
⑬ $500
⑭ 25%
⑮ $1\frac{3}{4}$ yrs.
⑯ 25%
⑰ $30
⑱ $1200
⑲ $990
⑳ 964, 6%
(21) $2,579

Grade 7b

Mathematics Objective

28. Use a compass and straightedge to:
    - a. bisect a segment
    - b. bisect an angle
    - c. copy an angle

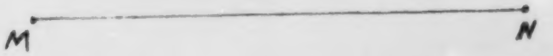
Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Bisect a Segment For teaching suggestions see page 134 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman) Bisect an angle p. 138 Copy an angle p. 139	
Bisect a Segment For teaching suggestions see page 218 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett) Bisect an Angle p. 231 Copy an Angle p. 228	8 (Silver Burdett) Copy an angle and bisect an angle p. 301-302 perpendicular line and parallel lines p. 303-304

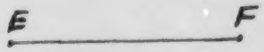
Name ______________________ Date ______________________

Period ______________________

Using only a compass and a straight edge construct each of the following. Show all the arcs and other construction lines used.

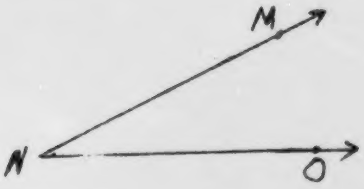
1. On line segment $\overline{MN}$ construct a line segment RS that is congruent to line segment $\overline{EF}$.



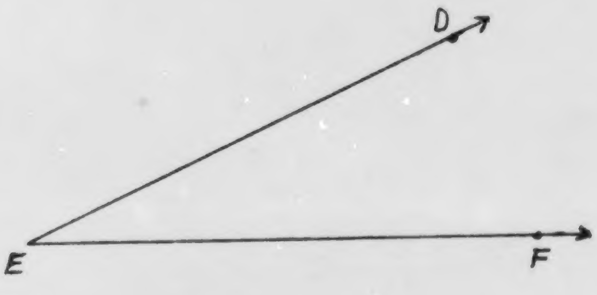


2. On line segment $\overline{CD}$ find the midpoint. Label it S.

3. Construct an angle <RST congruent to < MNO.



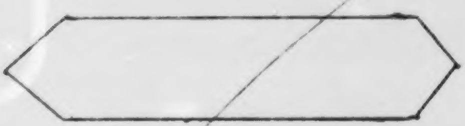
4. Bisect <DEF.



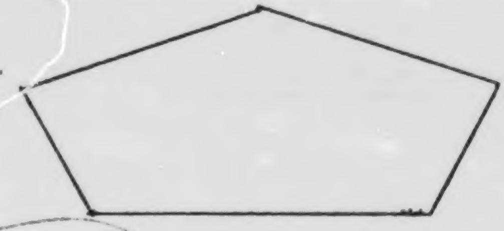
Name ______________________ Date ______________________

Draw each line or axis of symmetry. If a figure has none write the word none.

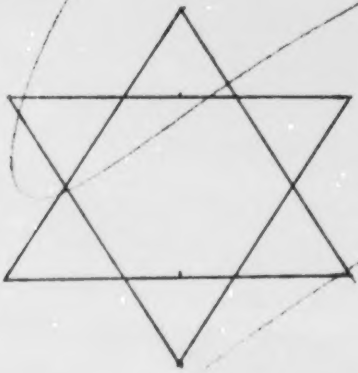
1.



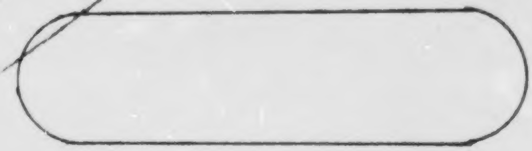
2.



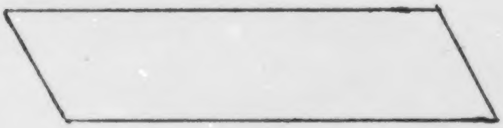
3.



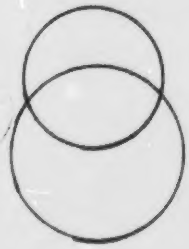
4.



5.



6.



Given the letters of the alphabet as shown below

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

answer the following

7. Fifteen of the letters have at least one axis of symmetry. Name them.

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

8. Name the letters having more than one axis of symmetry. ______________

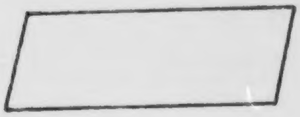
9. Name the letters which have a point of symmetry. ______________

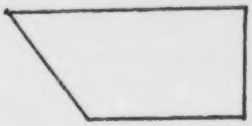
10. Name the letters which do not show symmetry.

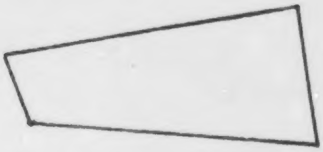
Name ______________________________ Date ______________________________

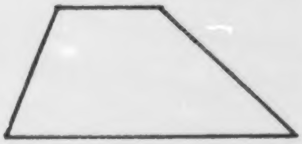
______________________________ Period ______________________________

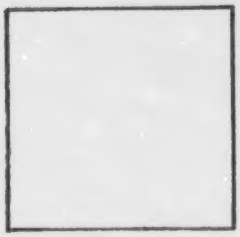
Construct a figure congruent to the given figure in the space provided.











Maintaining Skills

1. $$\frac{\left(\frac{5}{6} - \frac{3}{8}\right)}{3} \div \frac{\left(\frac{4}{5} \cdot \frac{2}{3} \cdot \frac{9}{16}\right)}{5} = \underline{\hspace{6cm}}$$

2. 104.9 + 96.176 + .9874 + 16 + 4.4 = ______________________

3. 63.97 x 8.004 = ______________

4. 64912 ÷ 732 = ________________ (fraction remainder)

5. 34% of 165 is what number? ________________

6. 24 is what percent of 66? ________________

7. 20% of what number is 17? ________________

8. $^2/_{30}$% = ________________ (fraction)

9. 164% = ________________ (fraction)

10. 1.6% = ________________ (decimal)

11. 54 = ________________ (percent)

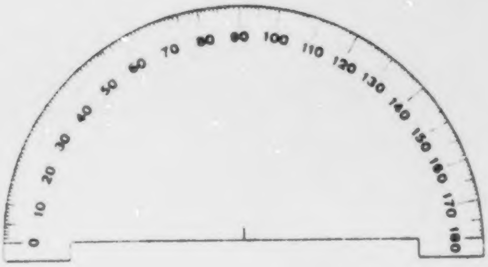
Answers:

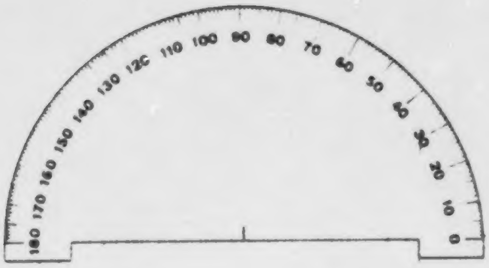
1. $2\,^{59}/_{108}$
2. 222.4634
3. 512.01588
4. $88\,^{124}/_{183}$
5. 56.1
6. $36\,^{4}/_{11}$
7. 85
8. $^{1}/_{1500}$
9. $^{41}/_{25}$
10. .016
11. 5400%

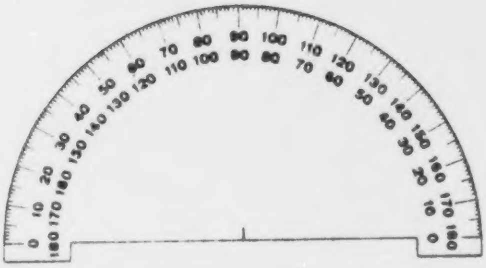
Mathematics Objective

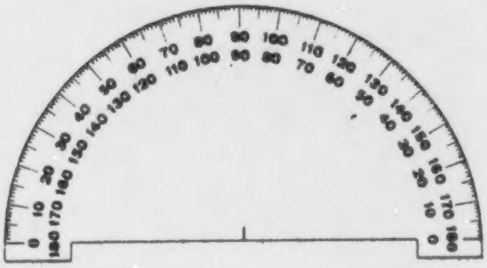
29M Use a protractor to measure an angle

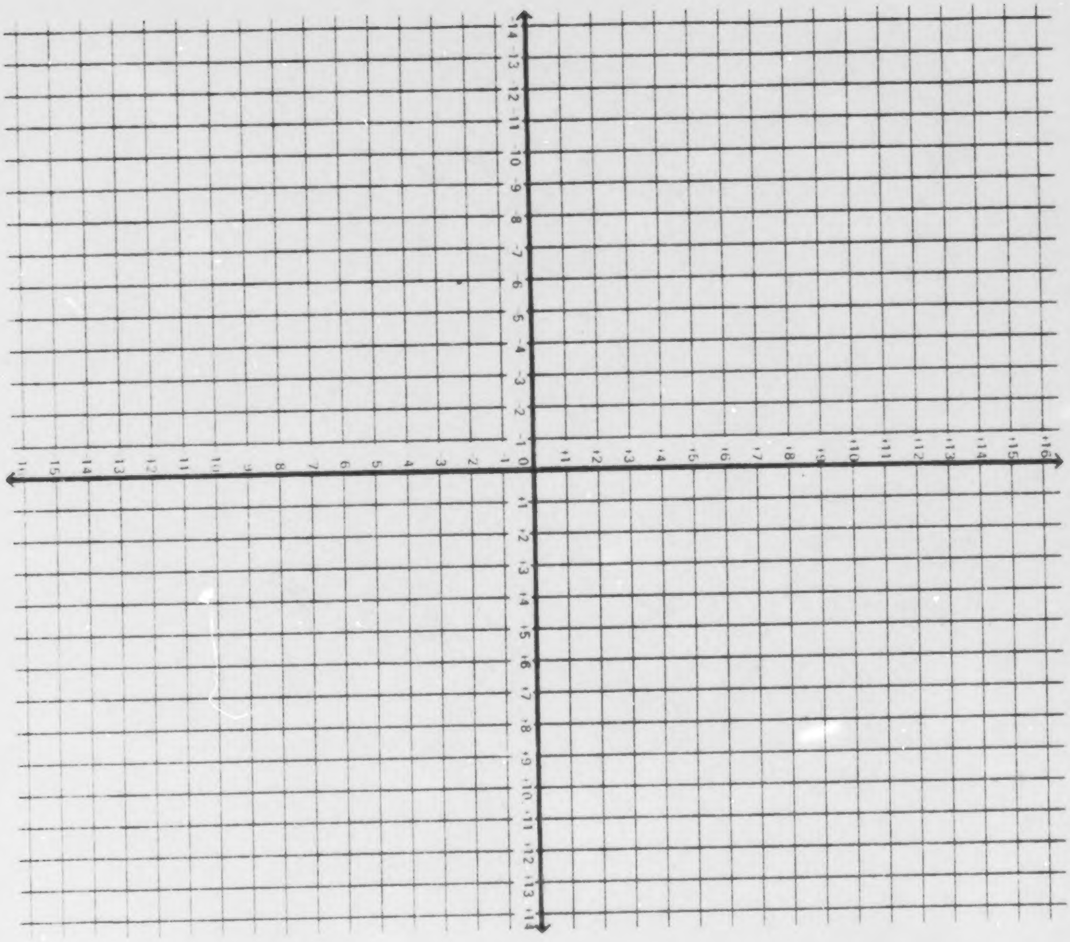
Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Measure angles For teaching suggestions see page 144 of the Teacher's Edition of *Mathematics Around Us* 7 (Scott Foresman) Classify Angles p. 144 Measure angles For teaching suggestions see page 222-223 of the Teacher's Edition of *Mathematics for Mastery* 7 (Silver Burdett) Classify angles p. 225	Angles of a parallelogram p. 200-201 8 (Scott Foresman) practice 7-57 7-L32 Classify angles p. 298-300 8 Silver Burdett Skillmaster 7 - 50 In Guide: 8:V:105:A Teacher Aide 6 Maintaining Skills MRC: MS F 11, 12, 13


0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180


180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0


0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0


0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0



15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0
+1 +2 +3 +4 +5 +6 +7 +8 +9 +10 +11 +12 +13 +14 +15 +16
14 13 12 11 10 9 8 7 6 5 4 3 2 1
+1 +2 +3 +4 +5 +6 +7 +8 +9 +10 +11 +12 +13 +14

Mathematics Objective

30a. Recognize that the sum of the angles of a triangle is 180 and use the knowledge to solve problems.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Sums of the measures of the angles of a triangle p. 202-208 8 (Scott Foresman)	practice 8 - 81 8 L 43, 44
Sum of the angles of a triangle p. 295 7 (Silver Burdett)	In Guide: Angles (2)
	MRC: PAWP: CC 8, 9, 10, 13, 14 MS: F 14
	Concepts of Geometry: Triangles Concepts of Geometry: Circles

Angles (2)

① An angle whose measure is greater than 90° but less than 180° is called a(n) ________________ angle.

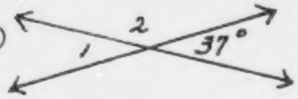
② An angle whose measure is less than 90° is called a(n) ________________ angle.

③ Two angles whose measures total 180° are called ________________________ angles.

④ Two angles whose measures total 90° are called ________________________ angles.

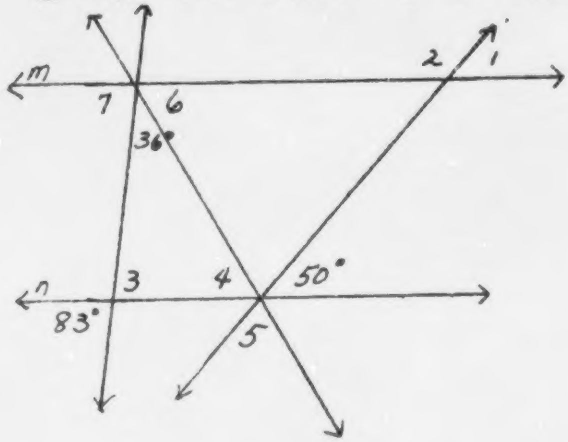
⑤ An angle whose measure is 90° is a ____________ angle.

⑥ If m(∠CBA) = 90°, then $\overleftrightarrow{BA}$ is ________________ to $\overleftrightarrow{BC}$.

⑦ 

The measure of ∠1 is _________ and the measure of ∠2 is _________.

⑧ The instrument used to measure angles is a ___________.



line m ∥ line n

⑨ m∠1 = _______

⑩ m∠2 = _______

⑪ m∠3 = _______

⑫ m∠4 = _______

⑬ m∠5 = _______

⑭ m∠6 = _______

⑮ m∠7 = _______

Answers

Angles

① $\angle ABC$, $\angle CBD$, $\angle ABD$
② $\overrightarrow{BA}$, $\overrightarrow{BC}$; $\overrightarrow{BC}$, $\overrightarrow{BD}$
③ $\overrightarrow{BC}$
④ No
⑤ $\angle RSU$ and $\angle USV$
$\angle USV$ and $\angle VST$
$\angle RSV$ and $\angle VSt$
$\angle RSU$ and $\angle UST$
$\angle MOP$ and $\angle POQ$
$\angle POQ$ and $\angle QON$
$\angle MOP$ and $\angle PON$
$\angle MOQ$ and $\angle QON$

⑥ Yes ⑦ No ⑧ $\angle 1$ and $\angle 3$

⑨ $\angle 1$ and $\angle 2$
$\angle 2$ and $\angle 3$
$\angle 3$ and $\angle 4$
$\angle 4$ and $\angle 5$
$\angle 5$ and $\angle 6$
$\angle 6$ and $\angle 1$

⑩ $\angle 1$ and $\angle 4$
$\angle 2$ and $\angle 5$
$\angle 3$ and $\angle 6$

① $\angle 1$ and $\angle 8$
$\angle 3$ and $\angle 5$
$\angle 2$ and $\angle 7$
$\angle 4$ and $\angle 6$

② $\angle 2$ and $\angle 5$
$\angle 4$ and $\angle 8$

③ $\angle 1$ and $\angle 6$
$\angle 3$ and $\angle 7$

④ $\angle 1$ and $\angle 2$
$\angle 1$ and $\angle 3$
$\angle 3$ and $\angle 4$
$\angle 2$ and $\angle 4$
$\angle 7$ and $\angle 8$
$\angle 4$ and $\angle 5$
$\angle 3$ and $\angle 6$

④ $\angle 2$ and $\angle 8$
$\angle 8$ and $\angle 5$
$\angle 5$ and $\angle 6$
$\angle 6$ and $\angle 7$
$\angle 1$ and $\angle 7$

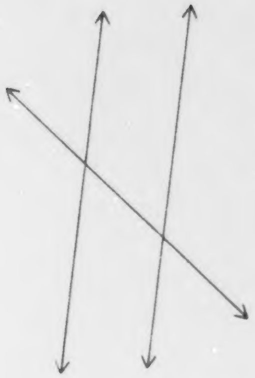
⑤ T ⑥ T ⑦ F ⑧ F
⑨ T ⑩ F

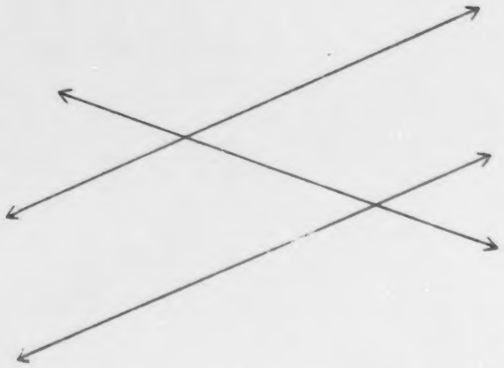
① obtuse
② acute
③ supplementary
④ complementary
⑤ right
⑥ perpendicular
⑦ 37° ; 143°
⑧ protractor
⑨ 50°
⑩ 130°
⑪ 83°
⑫ 61°
⑬ 69°
⑭ 61°
⑮ 83°

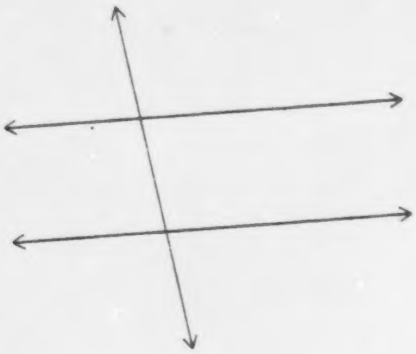
Mathematics Objective

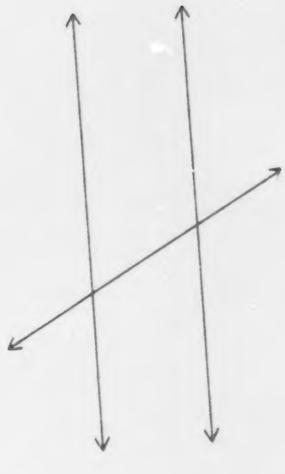
29. Find angles of equal measure when two parallel lines are cut by a transversal.
30. Identify pairs of angles having special sums.

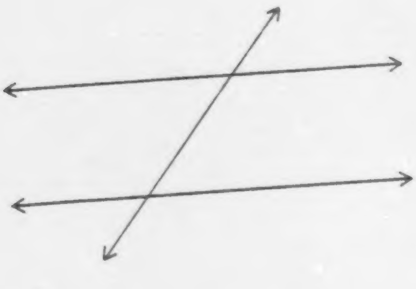
Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Parallel lines and transversals p. 196-201 8 (Scott Foresman)	In Guide: Teacher aid - parallel lines and transversals for transparency Angles (ans.)
Angles having special sums p. 299 8 (Silver Burdett)	Practice 8-42
Congruent angles formed by parallel lines p. 300 8 (Silver Burdett)	MRC: PAWP CC-8, 18 MS - F 14, F 11

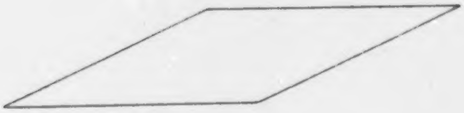


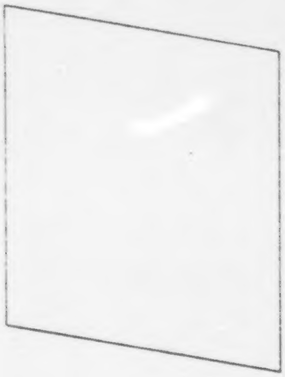


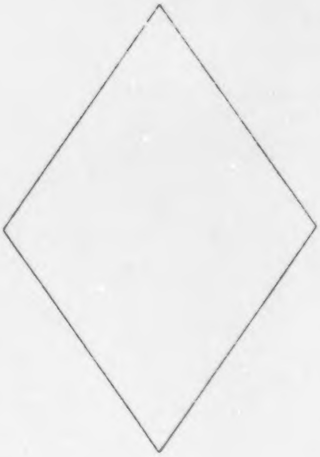


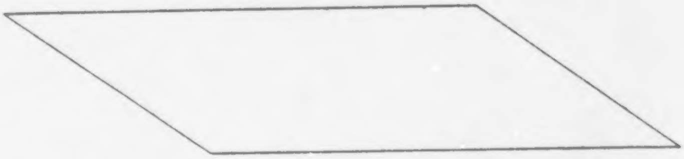




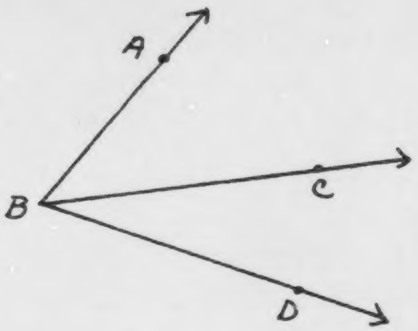








Angles



① Name the three angles in the drawing. ________

____

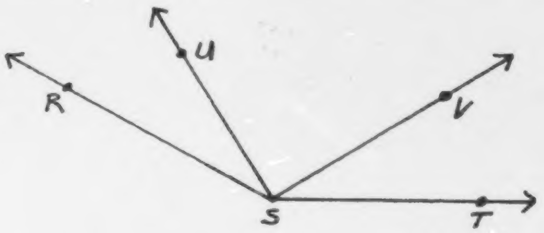
② Name the rays of ∠ABC, ________, ________ of ∠CBD, ________,

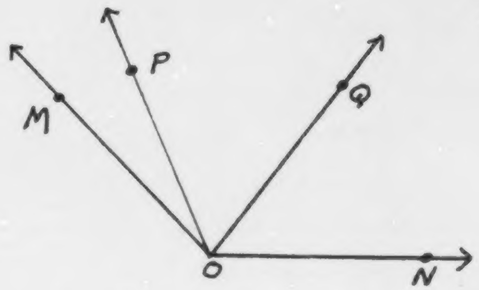
____

③ Which ray is common to both ∠ABC and ∠CBD? ________

④ Is ∠ABD adjacent to ∠CBD?

____

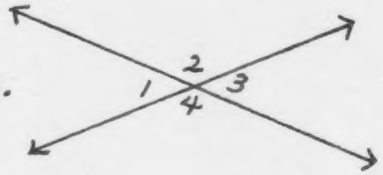




⑤ Name pairs of adjacent angles in each of the above drawings.

________ and ________ ________ and ________

________ and ________ ________ and ________



⑥ Are ∠1 and ∠2 adjacent?

⑦ Are ∠2 and ∠4 adjacent?

⑧ ∠2 and ∠4 are called vertical angles. Name another pair of vertical angles.

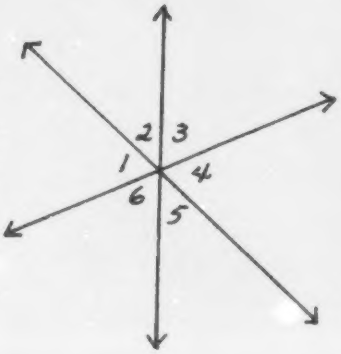
____

⑨ Name pairs of adjacent angles.

________ and ________ ________ and ________

________ and ________ ________ and ________

________ and ________ ________ and ________

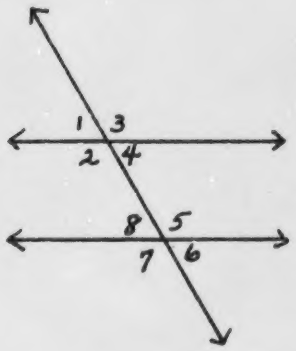


⑩ Name pairs of vertical angles.

________ and ________ ________ and ________

________ and ________

The figure shows parallel lines t and u cut by a transversal v.



① Name four pairs of corresponding angles.

____

② Name two pairs of alternate interior angles. ________________________

③ Name two pairs of alternate exterior angles. ________________________

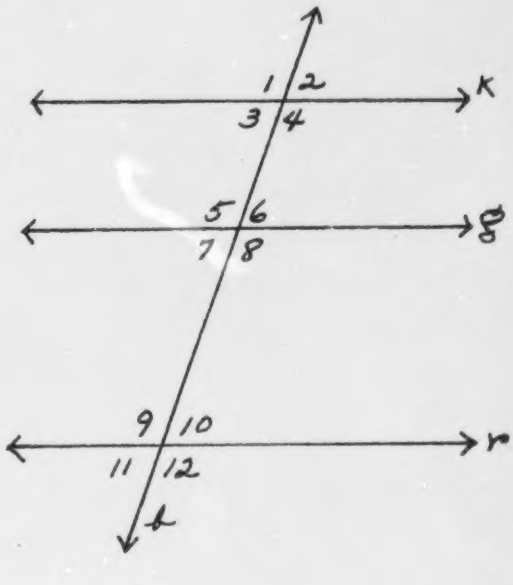
④ How many supplementary angles can you name? ______________________

____

____

____

Lines K ang g are parallel. $\angle 3 \cong \angle 11$. Decide whether the statements below are true or false.



⑤ $\angle 12 \cong \angle 1$

⑥ $r \parallel g$

⑦ $\angle 6 \cong \angle 8$

⑧ $m\angle 10 > m\angle 6$

⑨ $\angle 1 \cong \angle 4$

⑩ $r \parallel b$

Mathematics Objective

30b. Name corresponding parts of congruent polygons.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 210-214 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman) For teaching suggestions see page 306, 312-15 of the Teacher's Edition of *Mathematics Around Us* (Silver Burdett)	Practice 8-22 8-H 21 Skillmaster 8-71 In Guide: Ideas - Arithmetic Teacher, May, 1973 p. 372-3 Time Out (S.F.) 8:V:281:A 8:V:281:B 8:V:283:A 8:V:289:A MRC: WAP CC 18, CC 21 MS p. 9 Amusements in Mathematics in Development of Geometry Sine. pp. 28, 29, 30 Concepts of Geometry: Congruence andSimilarity

IDEAS

For Teachers

Objective: Experience in visualizing geometric objects with specific characteristics

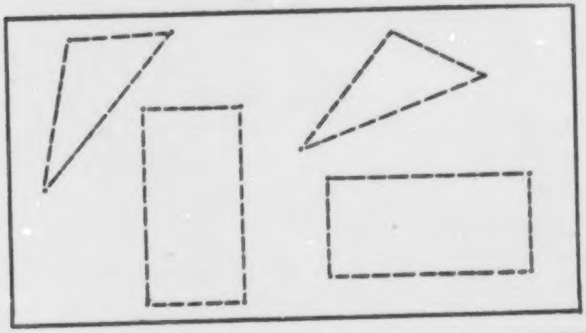
Levels: 6, 7, or 8

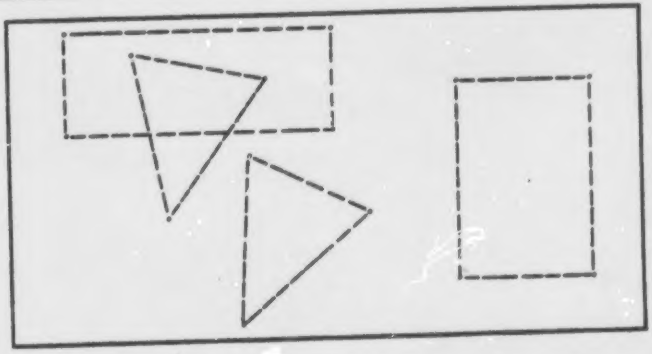
Directions for teachers:

1. Remove the activity sheet and reproduce a copy for each student.
2. Be sure that each student has a straightedge but do not require that he use it.
3. Have students work independently.

Comments: The activities can be made considerably more challenging by having your students try to find the congruent triangles *before* they draw the rectangles or by adding one or two extraneous points. The latter must be done very carefully so that other pairs of solutions are not introduced.

Key:





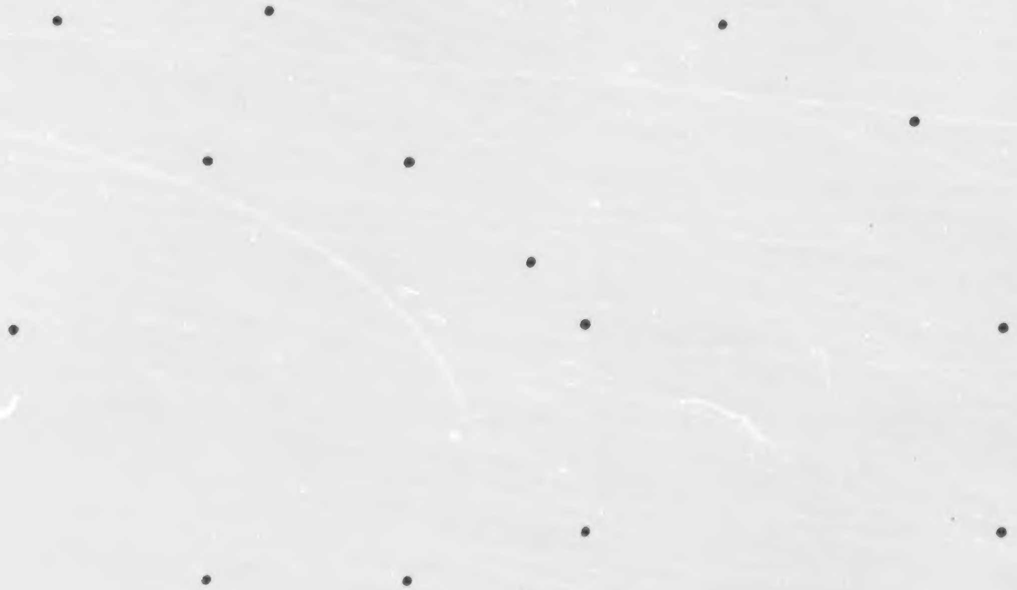
IDEAS

Name ______________

Use the points below as corners

Draw two congruent rectangles.

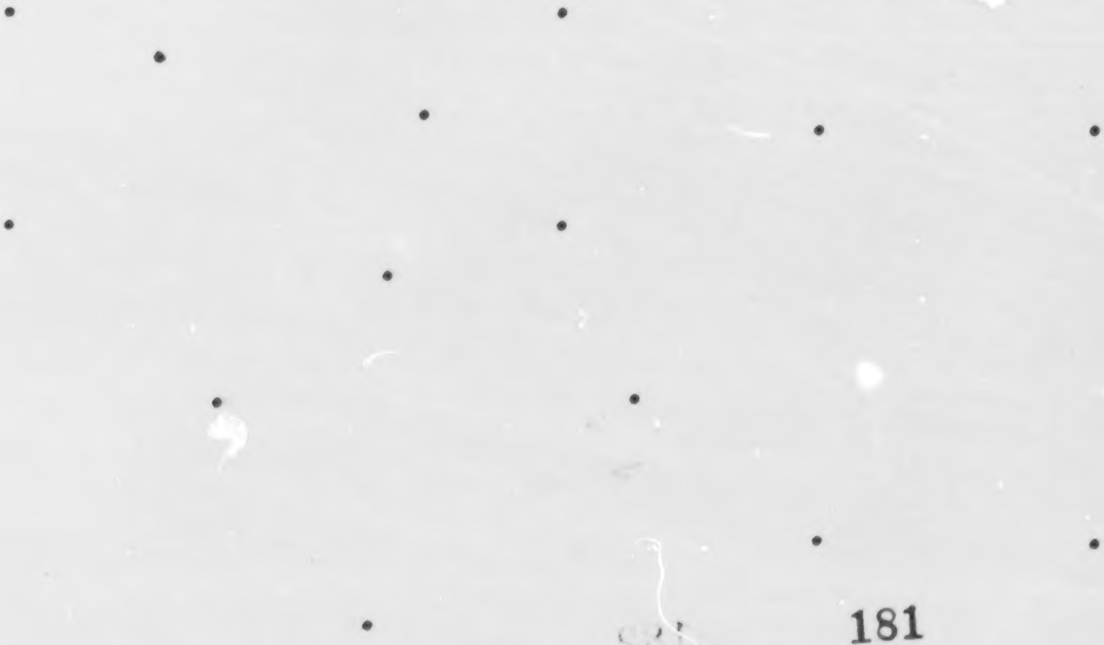
Draw two congruent obtuse triangles.



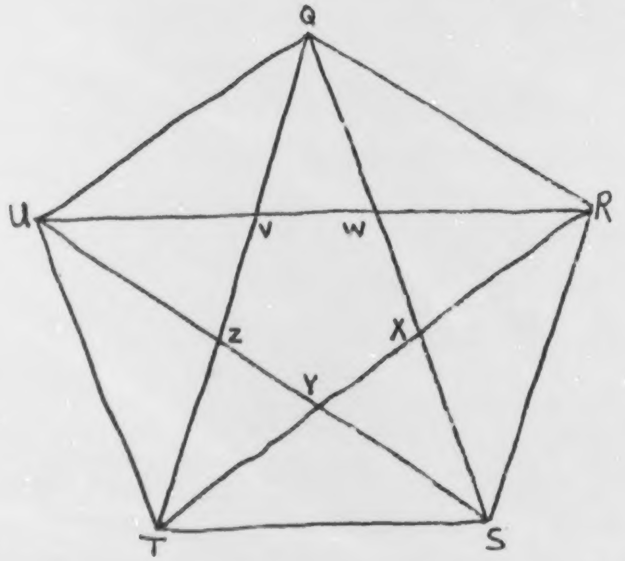
Use the points below as corners

Draw two rectangles with equal areas.

Draw two congruent acute triangles.



Time Out

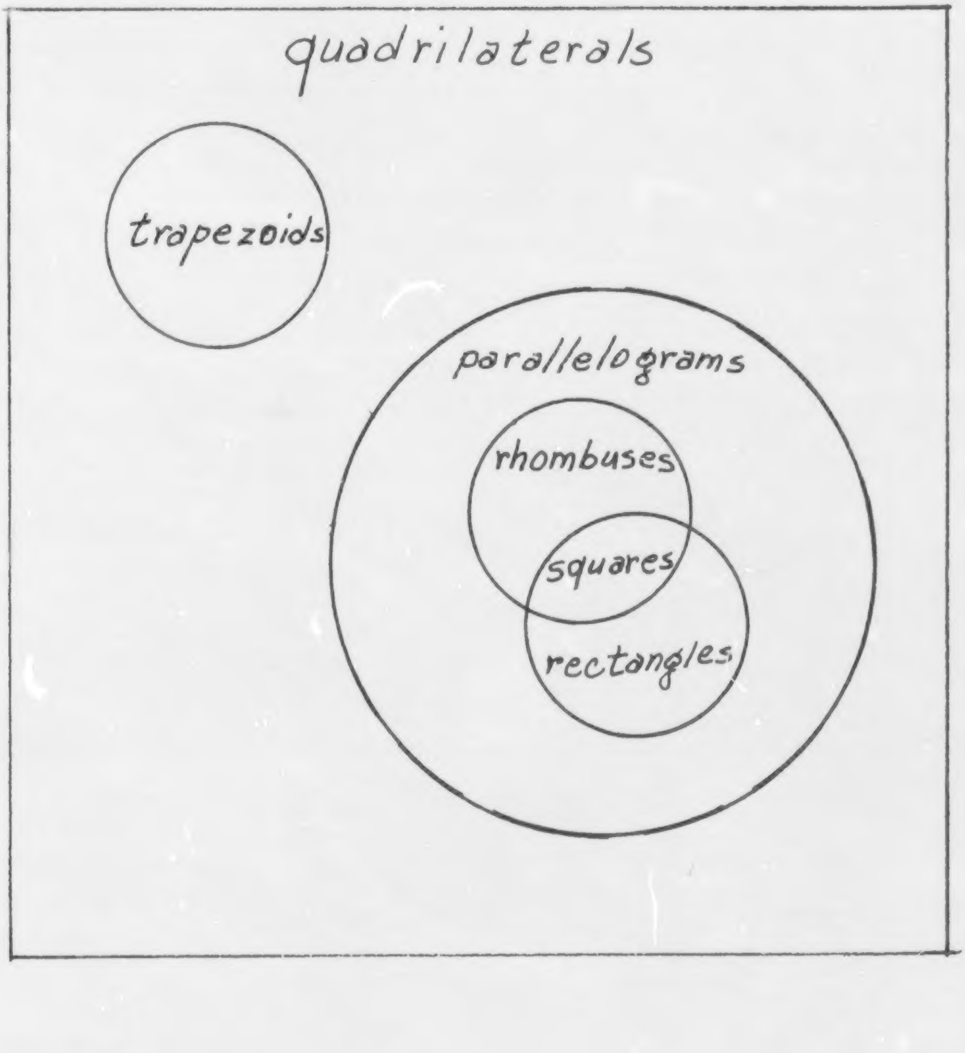


1. Find the four triangles that are congruent to triangle QVW.
2. Find the four triangles that are congruent to triangle QVU.
3. Find all the triangles that are congruent to triangle QRU.
4. Find all the triangles that are congruent to triangle QST.
5. Find all the triangles that are congruent to triangle URY.
6. Find all the triangles that are congruent to triangle QVR.

Answer: 1) UVZ, TYZ, SXY, RWX 2) TUZ, STY, RSX, QRW
3) QTU, STU, RST, QRS 4) QSU, RSU, RTU, QRT 5) OSZ, RTV, QTX, SUW
6) QUW, QUZ, TUV, STZ, STX, RSY, RS

Venn Diagram

Relationships among quadrilaterals

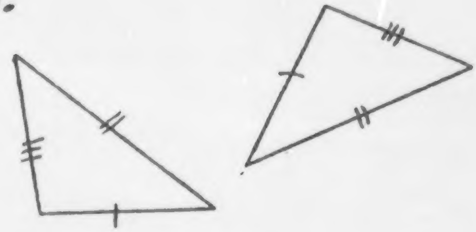

quadrilaterals
trapezoids
parallelograms
rhombuses
squares
rectangles

Name ______________________ Date ______________________

______________________ Period ______________________

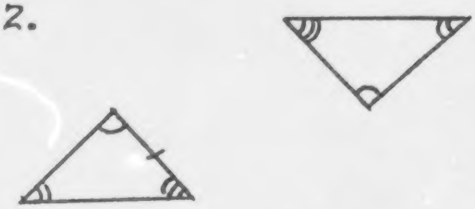
State the reason why each pair of triangles is congruent.

1.



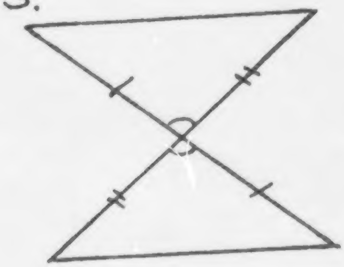
____

2.



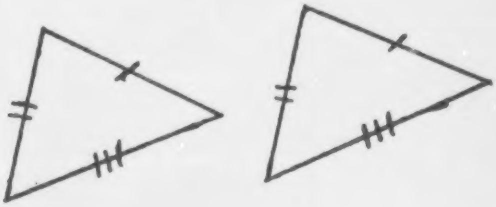
____

3.



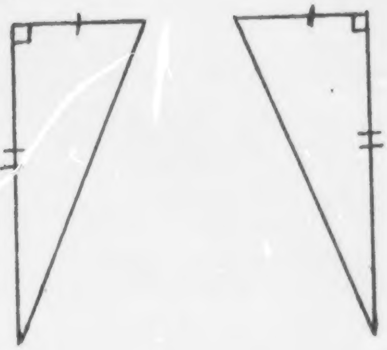
____

4.



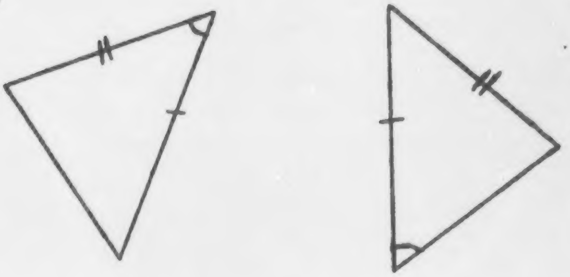
____

5



____

6.



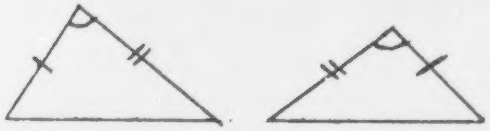
____

Name ______________________ Date ______________________

______________________ Period ______________________

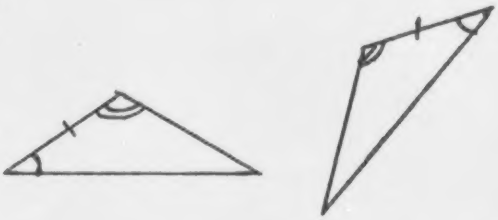
State whether the triangles in the given pairs are congruent by A. S. A.

1.



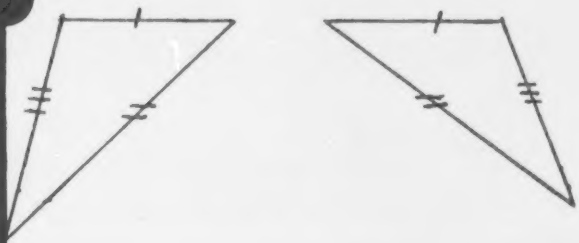
______ ______

2.



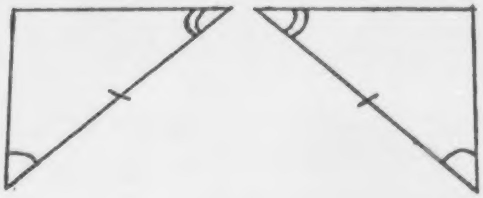
______ ______

3.



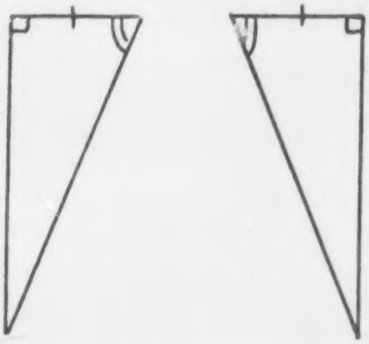
______ ______

4.



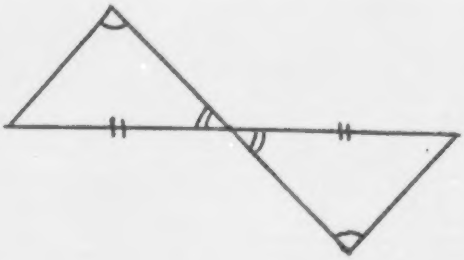
______ ______

5.



______ ______

6



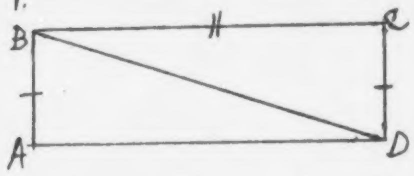
______ ______

Name ______________________ Date ______________________

______________________ Period ______________________

Name the congruent triangles shown in each figure and state why they are congruent.

1.



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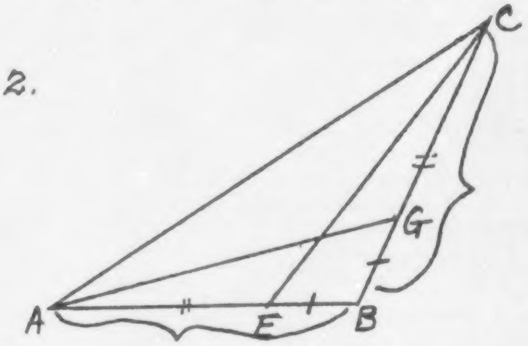

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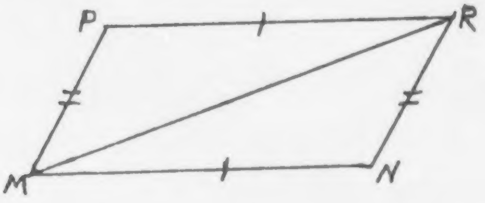

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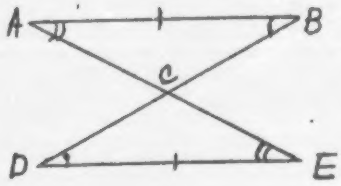

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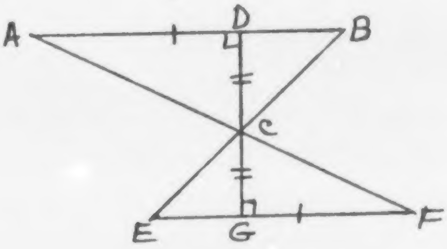

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activities

Edited by **Evan M. Maletsky,** *Montclair State College, Upper Montclair, New Jersey*
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COMPOUND INTEREST A LA SIMPLE CALCULATOR

By William D. Jamski, Indiana University Southeast, New Albany, IN 47150

Teacher's Guide

Grade Level: 8–12.

Materials: Student worksheets, one set of classroom transparencies, and at least one calculator for every two students.

Objectives: Students will explore the relationship between simple interest and compound interest by using calculators to develop inductively and intuitively the relationship

$$A = p\left(1 + \frac{r}{n}\right)^n .$$

Procedure: Before attempting this activity, students should be able to operate a four-function hand calculator. Each should be familiar with percent, the simple interest formula ($I = prt$), and the terms interest, principal, rate, period, and amount. After these prerequisite skills have been presented, sheet 1 should be distributed to check mastery. The notions of annually, semiannually, and quarterly are also developed here. After students have successfully finished sheet 1, sheet 2 can be distributed and completed under close teacher supervision. The second activity develops the compound interest concept. The effect of compounding is also graphically explored. With each activity appropriate calculator use should be encouraged. Sheet 3 presents several problems for determining the degree to which pertinent concepts have been learned.

Answers

Sheet 1: 1(d) 0.05, 0.025, 0.0125; 1(e) 0.0525, 0.02625, 0.013125; 1(f) 0.055, 0.0275, 0.01375; 1(g) 0.0575, 0.02875, 0.014375; 1(h) 0.06, 0.03, 0.015; 1(i) 0.0625, 0.03125, 0.015625; 1(j) 0.065, 0.0325, 0.01625; 2(a) 56.25; 2(b) 24.43; 2(c) 16.70; 2(d) 500; 2(e) 4½; 2(f) 180. *Sheet 2:* 115.76, 5.78(8), 121.54(8), $1.05^4 \times 100$; 121.54, 6.07(7), 127.61(7), $1.05^5 \times 100$; 127.61, 6.38(05), 133.99(05), $1.05^6 \times 100$; 133.99, 6.69(95), 140.68(95), $1.05^7 \times 100$; 140.68, 7.03(4), 147.7(14), $1.05^8 \times 100$. Each period's interest is added to the next period's principal. Not taking into consideration that the amounts of money were rounded down to the nearest penny,

$$A = p\left(1 + \frac{r}{n}\right)^n .$$

The graph for simple interest appears straight and crosses 200 at 10 years, while the graph of compound interest curves and crosses 200 at 7¼ years. *Sheet 3:* (1) \$862.50, (2) 6½%, (3) \$23.75 in interest is earned both ways. Over only one interest period, interest cannot be earned on interest. (4) \$5.93 + \$6.00 + \$6.07 + \$6.15 = \$24.15, \$24.15 − \$23.75 = 40¢, (5) 6.13%, $(1.015)^4 = 1.0614$, (6) 44 periods (11 years).

REVIEW OF BASIC CONCEPTS

Complete the following activities, using the calculator provided as needed. Scratch work may be done in the available space. Be sure to have questions answered before trying to go further.

1. Complete the following chart.

	Annual Rate	Equivalent Decimal	Semiannual Rate	Quarterly Rate
(a)	4 %	0.04	0.02	0.01
(b)	4½%	0.045	0.0225	0.01125
(c)	4 3/4%	0.0475	0.02375	0.011875
(d)	5%			
(e)	5¼%			
(f)	5½%			
(g)	5 3/4%			
(h)	6%			
(i)	6¼%			
(j)	6½%			

2. Taking into account the simple interest formula, I = prt (where I is the amount of interest, p is the principal, r is the annual rate, and t is the number of years or fractional part of a year), find each of the missing values. Where appropriate, consider 360 days as a business year. All dollar amounts are rounded down to the nearest cent.

	Interest	Principal	Annual Rate	Time
(a)	$______	$375.00	5 %	3 years
(b)	$______	$850.00	5 3/4%	6 months
(c)	$______	$495.00	4½%	270 days
(d)	$150.00	$_____	6 %	5 years
(e)	$45.00	$1500.00	___%	8 months
(f)	$37.50	$1200.00	6¼%	____ days

SHEET 2

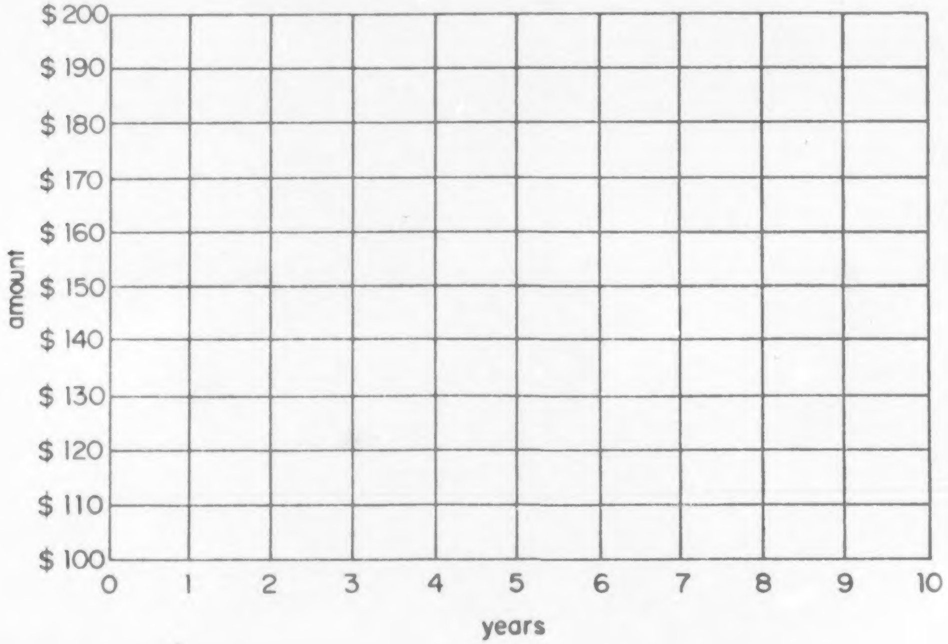
COMPOUNDING

1. Determine what happens to $100 compounded semiannually at 10% over 4 years. Round down to the nearest cent.

Principal	Semiannual Rate	Interest	Amount		
$100.00	½(10%)	$5.00	$105.00		$1.05^1 \times 100$
$105.00	0.05	$5.25	$110.25	end of year 1	$1.05^2 \times 100$
$110.25	0.05	$5.51(25)	$115.76(25)		$1.05^3 \times 100$
	0.05			end of year 2	$1.05^4 \times 100$
	0.05				
	0.05			end of year 3	
	0.05				
	0.05			end of year 4	

Describe two methods by which you can determine the amount of money available at the end of 5 years.

2. Using ●'s, graph what happens to $100 invested at 10% simple interest over a period of 10 years. Using x's, graph what happens to $100 compounded semiannually at 10% interest per year over a period of 10 years. Use your answers to exercise 1 above. Discuss the results.



SHEET 3

RELATED PROBLEMS

Using the results from sheets 1 and 2 and your calculator, as needed, solve the following problems.

1. Compute the simple interest earned on a principal of $3750 invested at 5 3/4% annually for a period of 4 years.

2. Compute the annual rate of interest needed to generate $19.50 in simple interest over a 3-year period on a principal of $100.00.

3. $500 is invested at 4 3/4% annually for 1 year. Compare the results of simple interest and compounding annually. Explain the results.

4. Using the figures in exercise 3, compare the difference in interest earned if compounded quarterly as opposed to simple interest.

5. An annual rate of 6% when compounded quarterly yields what rate of annual simple interest?

6. If a certain amount is invested at 6 1/2% compounded quarterly, how long will it take for the amount to double?

Mathematics Objective

31 Give the absolute value of a number and use

32 absolute value to compare integers

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
not in 8 (Scott Foresman)	not found in 7(Scott Foresman
absolute value For teaching suggestions see page 216 8 (Silver Burdett)	In guide: absolute value

Mathematics Objective

33 add and subtract pairs of integers

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
adding integers p. 50 - 51 8 (Scott Subtracting integers p. 52 - 54 8 (Scott	Integers - addition p. 366 - 369 7 (Scott Integers - Subtraction p. 370 - 371 7 (Scott
Integers - addition p. 218 - 223 Integers - Subtraction p. 224 - 228 8 (Silver -)	Integers - addition p. 371 - 374 7 (Silver -) Integers - Subtraction p. 375 - 376 7 (Silver -) In Guide: Integers - Addition and Subtraction MRC: AWA IV 6,8 PAWA AA13, AA20, AA23, AA63, AA65, AA66 Key to Algebra I pp. 8 - 23

Mathematics Objective

34 find the product and quotient of two integers

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Multiplying and dividing integers	Integers - Mult. & Division p. 372 - 73 7 (Scott –
For teaching suggestions see page 56, 58 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) p. 372 - 73 7 (Scott -	Integers - Mult. & Division p. 377 - 380 7 (Silver –)
For teaching suggestions see page 229, 233 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) p. 377 - 380 7 (Silver -)	SF. 8 practice 24, 25, 26 8 - L15 SB. Skillmaster 8 - 53, 54, 55 In Guide: Integers Multiplication & Division (ans) MRC: AWA (IV) 10, 12, 14 PWAP AA28 PWAP AA31 Key to algebra I 24 - 29

Integers

(Addition and Subtraction)

To add positive integers, add the same as whole numbers. The sum is positive

To add negative integers, add the same as whole numbers. The sum is negative.

To add a positive integer to a negative integer, subtract the absolute values and use the sign of the larger absolute value for the sum.

Positives	Negatives	Positive and Negative
$^{+}4 + ^{+}3 = ^{+}7$	$^{-}6 + ^{-}4 = ^{-}10$	$^{-}6 + ^{+}2 = ^{-}4$
$^{+}16 + ^{+}12 = ^{+}28$	$^{-}16 + ^{-}3 = ^{-}19$	$^{+}12 + ^{-}5 = ^{+}7$
$^{+}8 + ^{+}10 = ^{+}18$	$^{-}14 + ^{-}23 = ^{-}37$	$^{-}28 + ^{+}11 = ^{-}17$

To subtract one integer from another, add the additive inverse or opposite of the number being subtracted.

$^{+}4 - ^{+}7 = ^{+}4 + ^{-}7 = ^{-}3$

$^{-}10 - ^{-}8 = ^{-}10 + ^{+}8 = ^{+}2$

$14 - ^{-}16 = 14 + 16 = 30$

$^{-}6 - ^{+}18 = ^{-}6 + ^{-}18 = ^{-}24$

$^{-}3 - ^{-}7 = ^{-}3 + ^{+}7 = ^{+}4$

$20 - 40 = 20 + ^{-}40 = ^{-}20$

Complete the following: In the subtraction show the change to add the inverse before giving the difference.

① $^{+}16 + ^{-}9 =$

② $^{-}3 + ^{-}10 =$

③ $^{+}22 + ^{+}13 =$

④ $^{-}17 + ^{+}8 =$

⑤ $^{-}104 + ^{-}233 =$

⑥ $^{+}186 + ^{-}52 =$

⑦ $(^{-}17 + ^{+}10) + ^{-}4 =$

⑧ $(^{-}6 + ^{-}7) + ^{+}5 =$

⑨ $^{-}3 - ^{+}10 =$

⑩ $^{-}9 - ^{-}7 =$

⑪ $^{+}16 - ^{+}21 =$

⑫ $^{+}19 - ^{-}3 =$

⑬ $^{-}103 - ^{+}64 =$

⑭ $^{-}35 - ^{-}3 =$

⑮ $^{+}164 - ^{+}230 =$

⑯ $(^{-}17 - ^{-}21) - ^{+}10 =$

Integers

Multiplication and Division

If the signs of two integers are alike, their product is positive.

If the signs of two integers are different, their product is negative.

If the signs of two integers are alike, their quotient is positive.

If the signs of two integers are different, their quotient is negative.

Complete the following:

① $^{-}72 \div 8 =$

② $^{-}20 \div {}^{-}5 =$

③ $\frac{-12}{+6} =$

④ $^{+}3 \cdot {}^{-}12 =$

⑤ $^{-}56 \cdot {}^{-}16 =$

⑥ $^{+}4 \cdot {}^{+}16 =$

⑦ $^{-}4 \cdot {}^{+}5 \cdot {}^{-}6 =$

⑧ $^{-}100 \cdot {}^{-}100 \cdot {}^{-}3 \cdot {}^{-}4 =$

⑨ $^{-}52 \div {}^{+}13 \div {}^{-}2 =$

⑩ $\frac{0}{-16} =$

⑪ $^{+}64 \div {}^{-}8 \div {}^{+}2 =$

⑫ $^{-}144 \div {}^{-}12 \cdot {}^{+}6 \div {}^{-}9 =$

⑬ $\frac{(^{+}7)(^{-}8)}{^{-}4} =$

⑭ $^{+}56 \div (^{+}4 \cdot {}^{-}7) \div {}^{+}2 =$

⑮ $^{-}4 \cdot {}^{-}3 \cdot {}^{-}5 =$

⑯ $(^{+}3)(^{+}6) \div (^{-}3)(^{-}3) =$

⑰ $^{-}169 \div {}^{+}13 \cdot {}^{-}3 =$

⑱ $^{+}16 \cdot {}^{+}9 \div {}^{-}3 \div 12 =$

⑲ $^{-}54 \cdot {}^{+}6 \div {}^{-}2 \cdot {}^{-}5 =$

⑳ $^{-}18 \div {}^{-}2 \cdot {}^{-}3 \div {}^{-}9 =$

㉑ $^{+}32 \cdot {}^{-}4 \div {}^{-}16 \cdot {}^{+}3 =$

Answers

Absolute Value

① $3\frac{1}{2}$
② $\frac{7}{8}$
③ 1.41
④ $\frac{3}{7}$
⑤ $^{-}9$
⑥ 1.05
⑦ $^{-}5\frac{3}{4}$
⑧ 2.8

⑨ <
⑩ =
⑪ =
⑫ <

Integers - Add and Subtract

① $^{+}7$
② $^{-}13$
③ $^{+}35$
④ $^{-}9$
⑤ $^{-}337$
⑥ $^{+}134$
⑦ $^{-}11$
⑧ $^{-}8$
⑨ $^{-}13$
⑩ $^{-}2$
⑪ $^{-}5$
⑫ $^{+}22$
⑬ $^{-}167$
⑭ $^{-}32$
⑮ $^{-}66$
⑯ $^{-}6$

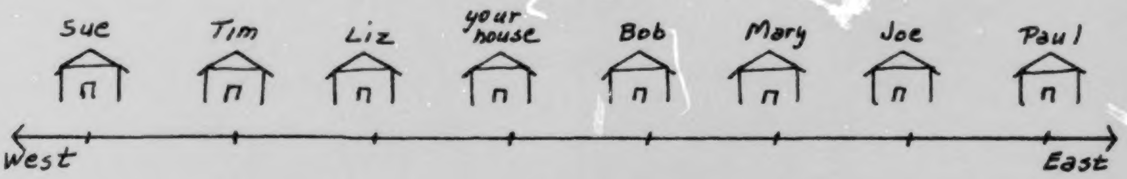
Integers - Mult. and Divide

① $^{-}9$
② 4
③ -2
④ $^{-}36$
⑤ 896
⑥ $^{+}64$
⑦ $^{+}120$
⑧ $^{+}120{,}000$
⑨ $^{+}2$
⑩ 0
⑪ $^{-}4$
⑫ $^{-}8$
⑬ $^{+}14$
⑭ $^{-}1$
⑮ $^{-}60$
⑯ $^{+}2$
⑰ $^{+}39$
⑱ $^{-}4$
⑲ $^{-}810$
⑳ $^{+}3$
㉑ $^{+}24$

ABSOLUTE VALUE

The greater of a rational number and it's opposite is the absolute value of the number. The absolute value of zero is zero.

When working with absolute value we consider only distance, disregarding direction. Consider visiting friends who live on your street. Lets assume all houses are the same distance apart.



If we let east be the positive direction, the west is the negative direction. Should you decide to visit Mary then visit Sue before returning home you would travel $^{+}2$ units, then $^{-}6$ units, then $^{+}4$ units. Adding this gives a sum of zero. Does that mean you did no walking? No, it illustrates why it is sometimes necessary to consider distance without considering direction. The absolute value of a number is always a positive number.

The absolute value of 5 is written $|5|$.

$|^{-}4| = 4$ $\quad |3.4| = 3.4$ $\quad ^{-}|^{-}5| = {}^{-}5$ $\quad ^{-}|6| = -6$

Find the absolute value of the following:

① $|^{-}3\frac{1}{2}| =$ ③ $|1.41| =$ ⑤ $^{-}|-9| =$ ⑦ $^{-}|5\frac{3}{4}| =$

② $|\frac{7}{8}| =$ ④ $|-\frac{3}{7}| =$ ⑥ $|^{-}1.05| =$ ⑧ $|^{-}2.8| =$

Use the symbel >, <, or = to complete the following:

⑨ $|-4| \bigcirc |6|$ ⑪ $|\frac{3}{4}| \bigcirc |-\frac{6}{8}|$ ⑫ $-|2| \bigcirc |-2|$

⑩ $8 \bigcirc |-8|$

Mathematics Objective

37 recognize examples for the use of properties of national numbers.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
S.F. none	
Properties of Rational Numbers	
SB. p. 252, 258	In Guide: Properties of Rational numbers. Properties
	MRC: PAWP AA 46
	PAWP AA 65
	AA 66
	AA 63
	Key to algebra Book I p. 29 amusements Mathematics in developing Geometry Series. p. I

Properties of Rational Numbers

Addition	Multiplication
Closure: $a + b$ is a rational number	Closure: $a \cdot b$ is a rational number
Commutative: $a + b = b + a$	Commutative $a \cdot b = b \cdot a$
Associative: $(a + b) + c = a + (b + c)$	Associative: $(a \cdot b)\, c = a \cdot (b \cdot c)$
Identity $a + 0 = 0 + a = a$	Identity $a \cdot 1 = 1 \cdot a = a$
Inverse (Opposite) $a + {}^{-}a = 0$	Inverse (reciprocal) $a \cdot \frac{1}{a} = 1$
Distributive: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$	

Properties

Identify the property shown in each of the following:

(1) $[(-3)4](^{-}5) = (-3)[4(^{-}5)]$ ________________

(2) ${}^{-}7 \cdot {}^{-}6 = {}^{+}42$ ________________

(3) $\frac{3}{8} + {}^{-}\frac{3}{8} = 0$ ________________

(4) ${}^{-}7 + (9 + {}^{-}1) = ({}^{-}7 + 9) + {}^{-}1$ ________________

(5) $(2 + 7) + 5 = 5 + (2 + 7)$ ________________

(6) ${}^{-}4(3 + {}^{-}8) = {}^{-}4 \cdot 3 + {}^{-}4 \cdot {}^{-}8$ ________________

(7) $-6(7^{0}) = {}^{-}6$ ________________

(8) ${}^{-}.8(.7) = .7({}^{-}.8)$ ________________

(9) $-\frac{2}{5} \cdot -\frac{5}{2} = 1$ ________________

(10) $7 + 10 = 17$ ________________

(11) $(4 \cdot 6) \cdot 7 = 4 \cdot (6 \cdot 7)$ ________________

(12) $\frac{5}{6} + {}^{-}\frac{1}{4}$ is a rational number ________________

(13) $(5 + {}^{-}4) + (3 + {}^{-}2) = (3 + {}^{-}2) + (5 + {}^{-}4)$ ________________

(14) $16 + 0 = 16$ ________________

(15) $6 + {}^{-}6 = 0$ ________________

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Mathematics Objective

38 find the sum, difference, product, or quotient of two given rational numbers.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
difinition of rational number	practice 8 - 61
For teaching suggestions see page 137 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)	In Guide: Rational numbers - definition and worksheet Rational number. (Worksheet No. 2) Rational numbers. (additional problems) answers
Identify & compare rational numbers - p. 240 - 247 add and subtract rational numbers	Skillmaster 8 - 57, 58
For teaching suggestions see page 250 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)	Skillmaster 8 - 59, 60, 61
Multiply and divide rational numbers	
For teaching suggestions see page 256 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)	

RATIONAL NUMBERS

A rational number is a number that can be expressed as the quotient of two integers; the divisor ay never be zero.

Example: $-4 = \frac{-4}{1}$ $\quad 4.13 = \frac{413}{100}$ $\quad -2\frac{1}{2} = -\frac{5}{2}$

$\frac{3}{-2}$ $\quad -5.1 = \frac{{}^-51}{10}$ $\quad 5 = \frac{5}{1}$

The rules for integers apply to rational numbers.

Solve the following:

① $({}^-56 \div 8) + {}^-7 =$
② $(13 - {}^-13) \div ({}^-14 \div 7) =$
③ $(5 \cdot {}^-4) - (26 - {}^-13) =$
④ $({}^-48 \div {}^-8) + ({}^-36 \div 6) =$
⑤ $(56 \div {}^-7) + ({}^-7 \cdot 7) =$
⑥ $-7\frac{1}{9} \div 5\frac{1}{3} =$
⑦ $-4\frac{1}{3} + 5\frac{1}{4} =$
⑧ $\frac{4}{5} - \frac{7}{5}$
⑨ $-\frac{3}{4} + \frac{1}{3} + -\frac{2}{3} =$
⑩ $7.6 + {}^-6.4 + {}^-1.3 + 2.5 =$
⑪ $-\frac{3}{5} + \frac{1}{4} - \frac{2}{3} =$
⑫ $1.4 + {}^-.5 =$
⑬ $3.4 - {}^-16.35 =$
⑭ $(-1)(-8)(16)(142)(0)(-2) =$
⑮ $-4\frac{2}{3} \cdot 5\frac{1}{4} \cdot {}^-1\frac{1}{2} =$
⑯ $-64\frac{1}{4} + 48\frac{2}{3} =$
⑰ $9\frac{3}{7} - {}^-3\frac{1}{2}$
⑱ $-.634 - 1.0934 =$
⑲ $-4.9 - {}^-5.6 =$
⑳ $-4.806 \div .12 =$
㉑ $(68.04)({}^-3.975) =$
㉒ $9\frac{1}{4} \div \frac{5}{6} =$
㉓ $-3\frac{1}{2} \div {}^-4\frac{2}{3} =$
㉔ $-1\frac{3}{4} \cdot 2\frac{2}{14} \cdot -\frac{5}{6} =$
㉕ $-\frac{15}{16} \cdot -\frac{24}{45} \cdot -\frac{9}{12} =$
㉖ $\frac{10}{27} \div \frac{-18}{35} \div \frac{-49}{54} =$

RATIONAL NUMBERS
(Work sheet #2)

Arrange in order from least to greatest

① $-\frac{7}{8}, \frac{7}{8}, -\frac{2}{3}, \frac{2}{3}, 0, 1, ^{-}1$

② $-\frac{1}{2}, -\frac{3}{4}, 1, 0$

③ $^{-}2, \frac{1}{7}, \frac{3}{8}, \frac{2}{9}$

④ $^{-}3, ^{-}4, 16, ^{-}1, 0, 1, 3$

Evaluate for the given values of the variables.

⑤ $4x - 2y + 3z$ if $x = 3, y = ^{-}2, z = 5$
⑥ $6a - 2b - 3c$ if $a = ^{-}1, b = 2, c = ^{-}3$

Complete the following:

⑦ $8 \div 2 = 8 \cdot$ ___

⑧ $7 \div \frac{1}{3} = 7 \cdot$ ___

⑨ $\frac{3}{1} \cdot$ ___ $= 1$

(10) $a \div \frac{b}{c} = a \cdot$ ___

(11) $^{-}76 - 34 =$ ___ $+ ^{-}34$

(12) $15 - 7 = 15 +$ ___

(13) $3 + ^{-}5 = 3 -$ ___

(14) $7 - 2 = 2 +$ ___

(15) $^{-}65 - 28 = ^{-}65 +$ ___

(16) $15 \cdot 3 = 15 \div$ ___

(17) $\frac{x}{21} = 6$; $x = 6$ ___

(18) $15a = 135$
$a = ($___$)($___$)$

(19) $\frac{a}{28.8} = .16$
$a = ($___$)($___$)$

(20) $4 \cdot 7 - 4 \cdot 2 = 4($___$)$

(21) $9(2 + n) =$ ___

(22) $-3(6+4) =$ ___ $+$ ___

Rational Numbers
(Additional Problems)

(1) $6\frac{3}{8} \cdot \frac{1}{34} \cdot 16 =$

(2) $6\frac{4}{7} \div 1\frac{1}{22} \div 1\frac{1}{10} =$

(3) $-4\frac{7}{15} - {}^{-}3\frac{1}{10} =$

(4) $8\frac{1}{4} - 2\frac{2}{5} =$

(5) ${}^{-}5 + \frac{5}{6} =$

(6) ${}^{-}2\frac{1}{3} \div {}^{-}1\frac{5}{6} =$

(7) $[({}^{-}25 \div 5) \div {}^{-}5] \cdot {}^{-}1 =$

(8) $(-8)^3 =$

(9) $\left(\frac{3}{} + \frac{1}{7}\right) \div \left(\frac{4}{9} + \frac{2}{5}\right) =$

(10) $\frac{7}{16} \cdot \frac{{}^{-}2}{3} \cdot \frac{48}{56} \cdot \frac{{}^{-}9}{14} =$

(11) ${}^{-}24 + {}^{-}58 + 78 + {}^{-}65 =$

(12) $({}^{-}\frac{5}{8} + 1) \div {}^{-}1\frac{1}{3} =$

(13) $(-21 - 8\frac{1}{2} - {}^{-}9) \cdot 3\frac{1}{3} =$

(14) $-\frac{5}{8} \cdot \frac{-104}{85} \cdot \frac{-68}{117} =$

(15) ${}^{-}13 - {}^{-}5\frac{2}{7} =$

(16) ${}^{-}2\frac{3}{11} - 9\frac{1}{33} =$

(17) $-\frac{1}{4} \div 1\frac{3}{9} \div {}^{-}12 =$

(18) $({}^{-}2 - 2\frac{1}{2}) \div (1\frac{1}{6} + 1\frac{1}{2}) =$

(19) ${}^{-}\frac{2}{3} \cdot {}^{-}\frac{4}{9} \cdot {}^{-}\frac{5}{8} \cdot {}^{-}\frac{3}{6} =$

(20) $\frac{2}{3} \div {}^{-}6 =$

(21) $6.34 - 7.1 + 5 + {}^{-}.89 =$

(22) $.3 - .7 =$

(23) $2.14 - 5.36 =$

(24) $4.6 - {}^{-}2.1 + {}^{-}5.7 - 6.2 =$

(25) $3.8 - 2.72 + .005 =$

(26) $5.994 \div {}^{-}8.1 =$

(27) ${}^{-}.123 \div {}^{-}.41 =$

(28) $.05198 \div .046 =$

(29) ${}^{-}.804 \div .012 =$

(30) $(42.6)(.0011) =$

(31) $5.7 - {}^{-}31.1 + {}^{-}4.9 - 4.3 =$

(32) $8.74 \div .0019 =$

(33) $({}^{-}.0053)(1.64) =$

(34) $({}^{-}37.1)({}^{-}.013) =$

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Answers

Rational Numbers

① -14
② -13
③ -59
④ 0
⑤ -57
⑥ $-1\frac{1}{3}$
⑦ $\frac{11}{12}$
⑧ $-\frac{3}{5}$
⑨ $-1\frac{1}{12}$
⑩ 2.4
⑪ $-1\frac{1}{60}$
⑫ $.9$
⑬ 19.75

⑭ 0
⑮ $36\frac{3}{4}$
⑯ $15\frac{7}{12}$
⑰ $12\frac{13}{14}$
⑱ -1.7274
⑲ $.7$
⑳ -40.05
㉑ -270.459
㉒ $11\frac{1}{10}$
㉓ $\frac{3}{4}$
㉔ $3\frac{1}{8}$
㉕ $-\frac{3}{8}$
㉖ $\frac{50}{63}$

Rational Numbers #2

① $-1, -\frac{7}{8}, -\frac{2}{3}, 0, \frac{2}{3}, \frac{7}{8}, 1$
② $-\frac{3}{4}, -\frac{1}{2}, 0, 1$
③ $-2, \frac{1}{7}, \frac{2}{9}, \frac{3}{8}$
④ $-4, -3, -1, 0, 1, 3, 16$
⑤ 21
⑥ -1
⑦ $\frac{1}{2}$
⑧ $\frac{1}{3}$
⑨ 3
⑩ $\frac{c}{b}$
⑪ -76
⑫ -7
⑬ 5
⑭ $2+3$
⑮ -28
⑯ $\frac{1}{3}$

⑰ $6 \cdot 21$
⑱ $135 \cdot \frac{1}{15}$
⑲ $(28.8)(.16)$
⑳ $(7-2)$

Rational Numbers (Additional problems)

① 3
② $5\frac{5}{7}$
③ $-7\frac{17}{30}$
④ $5\frac{17}{20}$
⑤ $-4\frac{1}{6}$
⑥ $1\frac{3}{11}$
⑦ -1
⑧ -512
⑨ $1\frac{4}{113}$
⑩ $\frac{9}{56}$
⑪ -74
⑫ $-\frac{9}{40}$
⑬ $-68\frac{1}{3}$
⑭ $-\frac{4}{9}$
⑮ $-7\frac{5}{7}$
⑯ $-11\frac{10}{33}$
⑰ $\frac{1}{64}$
⑱ $-1\frac{11}{16}$
⑲ $\frac{5}{54}$
⑳ $-\frac{1}{9}$
㉑ 3.35
㉒ $-.4$
㉓ -3.22
㉔ -5.2
㉕ 1.085

㉖ $-.74$
㉗ $.3$
㉘ 1.13
㉙ -67
㉚ $.04686$
㉛ $-.4$
㉜ 4600
㉝ $-.008692$
㉞ $.4823$

Mathematics Objective

35. Multiply by negative and positive powers of ten
36. Express large numbers using scientific notation

Teaching Techniques and Introductory Activities

Powers of ten
For teaching suggestions see page 70-79 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Scientific Notation
For teaching suggestions see page 80-83 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Powers of ten - Scientific notation
For teaching suggestions see page 142, 260-261 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

Negative and zero exponents
p. 408 410
8 (Silver Burdett)

Supplementary Activities and Resources

practice 8-34
8-L 22

Powers of ten
p. 108, 118
7 (Scott Foresman)

practice 8 - 35, 36
8 - L 23, 24

In Guide: Exponents
Practice 1, 2

Powers of ten
p. 188-190
7 (Silver Burdett)

MRC.
PWAP: BB 8
BB 9
BB 11
BB 12
BB 13
BB 24

No scientific notation in
7 (Silver Burdett)

Exponents

The rules for using positive exponents, negative exponents and zero exponents with 10 applies to any number which is not equal to zero.

$10^3 = 10 \times 10 \times 10 = 1000$

$4^3 = 4 \times 4 \times 4 = 64$

$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$

$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \times 10 \times 10} = .001$

$4^{-3} = \frac{1}{4^3} = \frac{1}{4 \times 4 \times 4} = \frac{1}{64}$

$\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = \frac{1}{\frac{1}{8}} = 8$

To multiply powers of the same base, add the exponents.

$10^3 \times 10^5 = 10^{3+5} = 10^8$

$4^2 \times 4^3 = 4^{2+3} = 4^5$

$2^5 \times 2^{-3} = 2^{5+{}^{-}3} = 2^2$

$10^{-3} \times 10^{-2} = \frac{1}{10^3} \times \frac{1}{10^2} = \frac{1}{10^{3+2}} = \frac{1}{10^5}$

$4^{-3} \times 4^{-2} = \frac{1}{4^3} \times \frac{1}{4^2} = \frac{1}{4^{3+2}} = \frac{1}{4^5} = \frac{1}{256}$

$\left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-2} = \left(\frac{1}{2}\right)^{-3+{}^{-}2} = \left(\frac{1}{2}\right)^{-5} = \frac{1}{\left(\frac{1}{2}\right)^5} = 32$

To divide powers of the same base, subtract the exponents.

$10^3 \div 10^5 = 10^{3-5} = 10^{-2}$

$4^2 \div 4^3 = 4^{2-3} = 4^{-1}$

$2^5 \div 2^{-3} = 2^{5-{}^{-}3} = 2^8$

$10^{-2} \div 10^{-4} = 10^{-2-{}^{-}4} = 10^2 = 100$

$4^{-3} \div 4^{-2} = 4^{-3-{}^{-}2} = 4^{-1} = \frac{1}{4}$

$\left(\frac{1}{2}\right)^{-3} \div \left(\frac{1}{2}\right)^{-2} = \left(\frac{1}{2}\right)^{-3-{}^{-}2} = \left(\frac{1}{2}\right)^{-1} = 2$

Write each product or quotient with one exponent:

(1) $3^4 \times 3^{-6} =$

(2) $\left(\frac{1}{4}\right)^{-5} \div \left(\frac{1}{4}\right)^{-2} =$

(3) $(^{-}8)^{-2} \times (^{-}8)^7 =$

④ $\frac{x^{-4}}{x^{-7}} =$

⑤ $2^5 \times 2^{-2} \times 2^4 \times 2^{-3} =$

⑥ $\left(\frac{4}{3}\right) \div \left(\frac{4}{3}\right)^{-1} =$

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EXPONENTS
(practice)

Express each of the following without exponents:

(1) $(\frac{1}{2})^2 \cdot (\frac{1}{2})^2 =$

(2) $2^2 \cdot 2^3 =$

(3) $10^{-4} =$

(4) $(\frac{2}{3})^2 \cdot (\frac{3}{4})^2 =$

(5) $5^0 \cdot 5^{-2} =$

(6) $(\frac{-1}{10})^5 =$

(7) $(\frac{3}{4})^6 \div (\frac{3}{4})^4 =$

(8) $(-3)^5 \div (-3)^3 =$

(9) $(\frac{1}{2})^3 + (\frac{3}{8})^2 =$

(10) $1,009^0 =$

(11) $(-\frac{2}{3})^{-2} =$

(12) $-[(\frac{2}{3})^{-2}] =$

(13) $(\frac{1}{10})^2 \cdot (\frac{5}{1})^2 =$

(14) $(\frac{3}{5})^{-3} \cdot (\frac{1}{15}) =$

(15) $(-\frac{2}{5})^{-2} \div (-\frac{2}{5})^{-4} =$

(16) $(-3)^{-3} =$

(17) $(\frac{1}{3})^{-2} + (\frac{1}{4})^{-2} =$

(18) $(-3)^2 + (-2)^{-2} =$

(19) $(\frac{1}{4})^{-2} + (\frac{1}{4})^2 =$

(20) $(\frac{1}{3})^{-2} - (\frac{1}{4})^{-2} =$

(21) $({}^-3)^2 - (-2)^{-2} =$

(22) $(\frac{-1}{4})^{-2} - (\frac{-1}{4})^2 =$

(23) $(\frac{1}{3} + \frac{1}{4})^2 =$

(24) $(\frac{1}{3} - \frac{1}{4})^2 =$

(25) $({}^-3 + {}^-2)^2 =$

(26) $(\frac{1}{3} + \frac{1}{4})^{-2} =$

(27) $\frac{1}{2^{-3}} =$

(28) $x^4 \cdot x^{-8} =$

(29) $6^{-3} \times 6^2 \times 6^0 =$

(30) $10^3 \times 10^{-5} \times 10^4 \times 10^0 =$

(31) $10^{-3} \div 10^{-6} \div 10^{-4} \div 10^5 =$

(32) $3^2 \cdot (-2)^2 =$

PRACTICE (1)

Write each decimal numeral as a power of ten.

1) $10{,}000 =$
2) $1{,}000{,}000 =$
3) $.01 =$
4) $.001 =$
5) $.00000001 =$
6) $100{,}000{,}000$

Write each power of ten as a decimal numeral.

1) $10^{-2} =$
2) $10^{-9} =$
3) $10^{4} =$
4) $10^{5} =$
5) $10^{-6} =$
6) $10^{-8} =$

Simplify each expression and express the result as a decimal numeral.

1) $(10^{3})(10^{-1}) =$
2) $10^{5} \cdot 10^{-2} =$
3) $10^{-1} \cdot 10^{-2} =$
4) $(10^{-2})(10^{-3}) =$
5) $10^{3} \div 10^{1} =$
6) $10^{1} \div 10^{3} =$
7) $10^{-3} \div 10^{5} =$
8) $10^{-4} \div 10^{3} =$
9) $\dfrac{10^{-3}}{10^{-4}} =$
10) $\dfrac{10^{6}}{10^{-4}} =$
11) $\dfrac{10^{-5}}{10^{-2}} =$
12) $(10^{2})\left(\dfrac{10^{5}}{10^{-4}}\right) =$
13) $(10^{7} \div 10^{-3})(10^{-1}) =$
14) $\left(\dfrac{1}{10} \cdot 10^{-5}\right) \div \dfrac{10^{0}}{10^{-8}} =$

PRACTICE (2)

Express each of the following in scientific notation

① 378,000,000 =
② .0000074 =
③ 179400000 =
④ 3914.162 =
⑤ .0012 =
⑥ 63.2 =
⑦ .00006 =

⑧ 940,000,000,000 =
⑨ .00000000043 =
⑩ 36000 =
⑪ .0542 =

Express each of the following as a decimal numeral

① $1.2 \times 10^{4} =$
② $3.4 \times 10^{-3} =$
③ $7.4 \times 10^{7} =$
④ $9.26 \times 10^{4} =$
⑤ $2.94 \times 10^{-5} =$
⑥ $4.7 \times 10^{2} =$
⑦ $7.2 \times 10^{-4} =$
⑧ $6.32 \times 10^{3} =$

⑨ $(1.49)(10^{-9}) =$
⑩ $(3.4)(10^{2}) =$
⑪ $10^{-6} =$
⑫ $(9.1)(10^{6}) =$
⑬ $(3)(10^{-3}) =$
⑭ $4(10^{4}) =$
⑮ $2 \cdot 10^{-3} + 5 \cdot 10^{-3} =$

Simplify:

① $\dfrac{10^{2} \cdot 10^{-4}}{10^{3}} \div \dfrac{10^{5} \cdot 10^{-3}}{10^{-2}} =$

② $\dfrac{6^{3} \cdot 6^{-2} \cdot 6^{4} \cdot 6}{6^{5} \cdot 6^{-1} \cdot 6} =$

③ $\dfrac{(3 \cdot 10^{-2})}{2 \cdot 10} \div \dfrac{(2 \cdot 10^{3})}{(3 \cdot 10^{-2})} =$

Practice (1)

(Key)

1) 10^5
2) 10^6
3) 10^{-2}
4) 10^{-3}
5) 10^{-8}
6) 10^8

1) .01
2) .000000001
3) 10000
4) 100000
5) .000001
6) .00000001

Exponents (Answers)

① 3^{-2}
② $\left(\frac{1}{4}\right)^{-3}$
③ $(-8)^5$
④ x^3
⑤ 2^4
⑥ $\left(\frac{4}{3}\right)^2$

1) $10^2 = 100$
2) $10^3 = 1000$
3) $10^{-3} = .001$
4) $10^{-5} = .00001$
5) $10^2 = 100$
6) $10^{-2} = .01$
7) $10^{-8} = .00000001$
8) $10^{-7} = .0000001$
9) $10^1 = 10$
10) $10^{10} = 10,000,000,000$
11) $10^{-3} = .001$
12) $10^{11} = 100,000,000,000$
13) $10^9 = 1,000,000,000$
14) $10^{14} = .0000000000001$

PRACTICE (2)

1) 3.78×10^8
2) 7.4×10^{-6}
3) 1.79463×10^8
4) 3.91×10^3
5) 1.2×10^{-3}
6) 6.32×10
7) 6×10^{-5}
8) 9.4×10^{11}
9) 4.3×10^{-9}
10) 3.6×10^4
11) 5.42×10^{-2}

1) 12000
2) .0034
3) 74,000,000
4) 92,600
5) .0000294
6) 470
7) .00072
8) 6320
9)
9) .00000000149
10) 340
11) .000001
12) 9100,000
13) .003
14) 40000
15) .007

1) 10^{-9}
2) 6
3) 2.25×10^{-8}

Answers

Exponents (practice)

① $\frac{1}{16}$
② 32
③ .0001
④ $\frac{1}{4}$
⑤ $\frac{1}{25}$
⑥ .0001
⑦ $\frac{9}{16}$
⑧ 9
⑨ $\frac{17}{64}$
⑩ 1
⑪ $2\frac{1}{4}$
⑫ $-2\frac{1}{4}$
⑬ $\frac{1}{4}$
⑭ $\frac{25}{81}$
⑮ $\frac{4}{25}$
⑯ $-\frac{1}{27}$
⑰ 25
⑱ $9\frac{1}{4}$
⑲ $16\frac{1}{16}$
⑳ -7
㉑ $8\frac{3}{4}$
㉒ $15\frac{15}{16}$
㉓ $\frac{49}{144}$
㉔ $\frac{1}{144}$
㉕ 25
㉖ $2\frac{46}{49}$
㉗ $\frac{1}{8}$
㉘ x^{-4}
㉙ $6^{-1} = \frac{1}{6}$
㉚ 100
31 100
㉜ 36

Mathematics Objective

39. Choose an appropriate metric unit to measure mass, length, and capacity
40. Measure volume using liter and milliliter

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Metric Units in volume capacity and weight p. 109-119 8 (Scott Foresman)	Metric units in volume capacity and weight p. 68-86 7 (Scott Foresman)
7 practice 29, 35, 36 8-L31	
Metric Units p. 44-61 8 (Silver Burdett)	Metric Units p. 86-101 7 (Silver Burdett)
converting metric to metric units 130-31	Converting metric to metric units p. 203-204
Metric units of volume, capacity and weight p. 342-344 8 (Silver Burdett)	Metric units in volume, capacity and weight p. 326 7 (Silver Burdett)
Skill master 8-12, 13, 14, 15, 16	

Mathematics Objective

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Units of Area For teaching suggestions see page 196 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett) Units of volume, capacity and weight For teaching suggestions see page 336, 342 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)	In Guide: Metric conversion chart "Ideas" taken from *The Arithmetic Teacher*, May, 1972, p. 371-372 gives the student experience in converting from the metric system to the English system and vice versa. MRC: PWAP: CC 24, 25

METRIC CONVERSION CHART

multiply by 10 →

M			Km	hm	dam	m	dm	cm	mm			μ

← divide by 10

multiply by 10^2 →

M^2			Km^2	hm^2	dam^2	m^2	dm^2	cm^2	mm^2			μ^2

← divide by 10^2

multiply by 10^3 →

M^3			Km^3	hm^3	dam^3	m^3	dm^3	cm^3	mm^3			μ^3

← divide by 10^3

Move the decimal point in the direction of the arrow as many places as there are steps to the unit you are converting to.

1000 Kg = 1 t (metric ton)

$1\ dm^3 = 1\ l$

$1\ cm^3$ (cc) = 1 mL

1 liter of water weighs 1 Kilogram

1 mL of water weighs 1 gram

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Metric Conversion

① 6.4 km = ____________ m

② 5 m^2 = ____________ Km^2

③ .3 cm = ____________ hm

④ .05 mm = ____________ µ

⑤ 3.8 Km = ____________ dam

⑥ 7 dm^2 = ____________ cm^2

⑦ 1.5 m^2 = ____________ cm^2

⑧ .34 mm^2 = ____________ m^2

⑨ 154 ml = ____________ ℓ

⑩ 7.8 Kg = ____________ g.

⑪ 3.46 ℓ = ____________ ml

⑫ 6.3 t = ____________ Kg

⑬ 50.7 m^3 = ____________ cm^3

⑭ 7000 000 cm^3 = ____________ m^3

⑮ 13 cc = ____________ mm^3

16 6 ℓ of water = ____________ g

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⑰ .7 ml of water = ______________ Kg

⑱ 16 dm^3 = ______________ l

⑲ 1 cm^3 = ______________ dm^3

⑳ 600 cc = ______________ l

㉑ 10. 7 dm^3 = ______________ cc

㉒ 1.004 l = ______________ ml

㉓ .005 Km = ______________ mm

㉔ 6 mm = ______________ dam

㉕ 6 mm^2 = ______________ dam^2

㉖ 6 mm^3 = ______________ dam^3

㉗ 100 cc = ______________ ml

㉘ 6 Kg = ______________ t

㉙ 7 t = ______________ g

㉚ 4 mm^3 = ______________ m^3

Answers.

Metric Conversions

① 6400
② .000005
③ .00003
④ 50
⑤ 380
⑥ 700
⑦ 15000
⑧ .00000034
⑨ .154
⑩ 780
⑪ 3460
⑫ 6300
⑬ 50,700,000
⑭ 7
⑮ 13000
⑯ 6000
⑰ .0007
⑱ 16
⑲ .001
⑳ .6
㉑ 10700
㉒ 1004
㉓ 5000
㉔ .0006
㉕ .00000006
㉖ .000000000006
㉗ 100
㉘ .006
㉙ 7000000
30) .000000004

IDEAS

For Teachers

Objective: Experience in using conversion tables: English system to metric system

Grade levels: 6, 7, or 8

Directions for teachers:

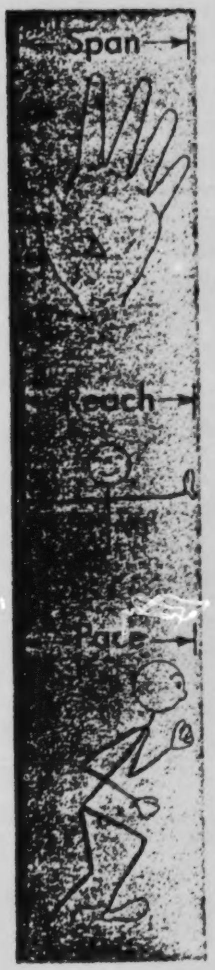
1. Reproduce for each student a copy of the Personal Data Sheet and of the conversion tables on the following page.
2. Place these pages on the activity table along with a tape measure, several rulers, and a bathroom scale.
3. Encourage students to fill out their Personal Data Sheet as an individual project. Allow a week for this "extra."

Comments: Some students may be sensitive about making "public" some of their personal data. You may avoid unpleasantness by allowing students to skip any data they consider too personal. Sorry, it is impossible to provide answers for the Personal Data Sheet.

IDEAS

Use your conversion tables to help you complete this sheet.

PERSONAL DATA SHEET



Name: ______________ Date: ____________

Age: __________ years ________ months

1. Height: ________ feet ________ inches

________ centimeters

________ meters

2. Weight: ________ pounds

________ kilograms

________ grams

3. Waist: ________ inches ________ centimeters

4. Chest: ________ inches ________ centimeters

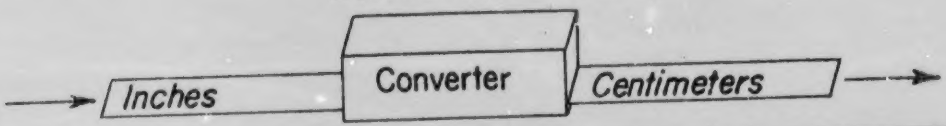
5. Span: ______ inches; ______ centimeters

6. Reach: ______ inches; ______ centimeters

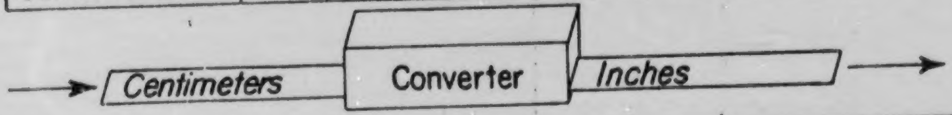
7. Pace: ______ inches; ______ centimeters

8. Length of shoe: ______ inches; ______ centimeters

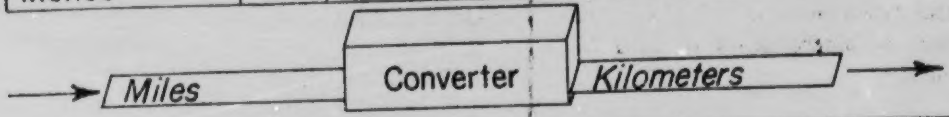
CONVERSION TABLES


Inches
Converter
Centimeters

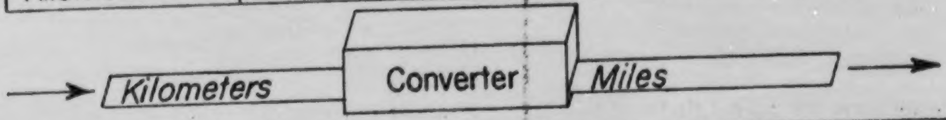
Inches	1	2	3	4	5	6	7	8	9
Centimeters	2.54	5.08	7.62	10.16	12.70	15.24	17.78	20.32	22.86


Centimeters
Converter
Inches

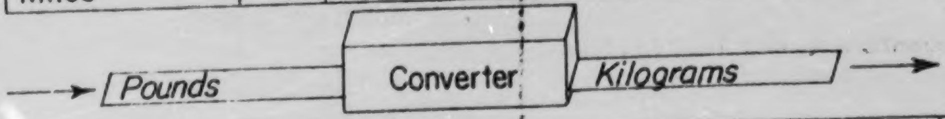
Centimeters	1	2	3	4	5	6	7	8	9
Inches	.39	.79	1.18	1.57	1.97	2.36	2.76	3.15	3.54


Miles
Converter
Kilometers

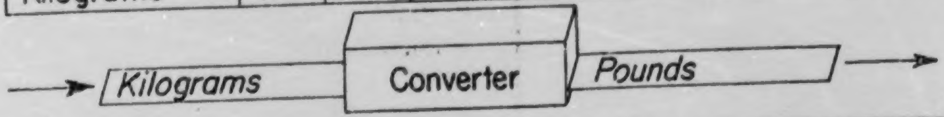
Miles	1	2	3	4	5	6	7	8	9
Kilometers	1.61	3.22	4.83	6.44	8.05	9.66	11.27	12.87	14.48


Kilometers
Converter
Miles

Kilometers	1	2	3	4	5	6	7	8	9
Miles	.62	1.24	1.86	2.49	3.11	3.73	4.35	4.97	5.59


Pounds
Converter
Kilograms

Pounds	1	2	3	4	5	6	7	8	9
Kilograms	.45	.91	1.36	1.81	2.27	2.72	3.18	3.63	4.08


Kilograms
Converter
Pounds

Kilograms	1	2	3	4	5	6	7	8	9
Pounds	2.20	4.41	6.61	8.82	11.02	13.23	15.43	17.64	19.84

Mathematics Objective

41. Change any given metric unit of length from one to another and any metric unit of area, volume, or mass from one to another

Teaching Techniques and Introductory Activities

Units of length
For teaching suggestions see page 108-109 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Units of area
For teaching suggestions see page 110-111 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Units of volume
For teaching suggestions see page 112 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Units of mass
For teaching suggestions see page 115-16 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Units of length
For teaching suggestions see page 130-133 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

Supplementary Activities and Resources

8 practice 49-56

Units of length
p. 68-74
7 (Scott Foresman)

Units of Area
p. 76
7 (Scott Foresman)

Units of Volume
p. 77-79
7 (Scott Foresman)

Units of Mass
p. 80-81
7 (Scott Foresman)

Units of length
p. 94-95
7 (Silver Burdett)

Units of Area
p. 302
7 (Silver Burdett)

Units of Volume
p. 100
7 (Silver Burdett)

Units of Mass
p. 96-99
7 (Silver Burdett)

Skillmaster 8-36

Mathematics Objective

42. Name the greatest possible error using precision in linear measurement
43. Give the number of significant digits in a given measure

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Precision, g.p.e. For teaching suggestions see p. 120-121 8(Scott Foresman)	practice 8-57
Significant digits For teaching suggestions see p. 122-123 8 (Scott Foresman)	practice 8-58 8-H14 8-H13
Precision, g.p.e. For teaching suggestions see p. 134-137 8 (Silver Burdett)	Precision For teaching suggestions see p. 151-155 7 (Silver Burdett)
Significant digits For teaching suggestions see p. 139 8 (Silver Burdett)	In Guide: Greatest Possible Error Worksheet Maintaining skills

Greatest Possible Error

The unit of measurement is the smallest unit used in naming the measure.

measure	unit of measure
$\frac{3}{4}$ yd	$\frac{1}{4}$ yd
.67 cm	.01 cm
$3\frac{1}{2}$ in.	$\frac{1}{2}$ in.
6.1 m	.1 m

The greatest possible error of the measurement is one-half of the unit of measure.

measure	unit of measure	g.p.e.
$\frac{3}{4}$ yd.	$\frac{1}{4}$ yd.	$\frac{1}{8}$ yd.
.86 cm	.01 cm	.005 cm
$4\frac{1}{2}$ in.	$\frac{1}{2}$ in.	$\frac{1}{4}$ in.
6.1 m	.1 m	.05 m

Complete this table

	measure	unit of measure	greatest possible error
①	$10\frac{3}{8}$ yd.		
②	7.4 m		
③	$3\frac{2}{3}$ mi.		
④	$5\frac{1}{2}$ ft.		
⑤	17.04 cm		
⑥	16 m		
⑦	9.76 cm.		
⑧	.012 Km		

Maintaining Skills

① Is 8544 divisible by 2 ____, 3 ____, 4 ____,
5 ____, 6 ____, 8 ____, 9 ____, or 10 ____?

② Is 91 prime or composite?

③ Name the prime factorization of 225 using exponents.

④ Rename $416_{(8)}$ in base ten.

⑤ Write 905,000,050.914 in words.

⑥ $\left[\dfrac{2(7+3\times 2)}{3\times 3+4}\right]\left[\dfrac{24\div 3+4}{10\div 5}\right]=$

⑦ $\dfrac{\frac{3}{4}(5+7)}{5+4}\cdot\dfrac{5+3\div 5-2}{32\div 4-1}=$

⑧ Name the GCF and LCM of 36, 90, 18

⑨ $\frac{3}{7}=$ ________ (percent)

⑩ .0047 = ______ (percent)

⑪ 3.4% = ________ (fraction)

⑫ 96.8% = _______ (decimal)

⑬ 6% of what number is 18? ________

⑭ 15 is what percent of 37? _______

⑮ 14% of 64 is what number? _______

Answers

Greatest Possible Error

① $\frac{1}{8}$ yd $\frac{1}{16}$ yd
② .1 m .05 m
③ $\frac{1}{3}$ mi $\frac{1}{6}$ mi.
④ $\frac{1}{2}$ ft $\frac{1}{4}$ ft.
⑤ .01 cm .005 cm
⑥ 1 m .5 m
⑦ .01 cm .005 cm
⑧ .001 Km .0005 Km

Maintaining Skills

① yes, yes, yes, no, yes, yes, yes, no.
② composite
③ $3^2 \cdot 5^2$
④ 270
⑤ Nine hundred five million fifty and nine hundred fourteen thousandths.
⑥ 12
⑦ $2\frac{4}{7}$
⑧ GCF = 18
LCM = 180
⑨ $42\frac{6}{7}\%$
⑩ 47 %
⑪ $\frac{17}{500}$
⑫ .968
⑬ 300
⑭ $40\frac{30}{37}\%$
⑮ 8.96

Mathematics Objective

44M identify equations and inequalities as open number sentences or statements.

44 recognize equivalent sentences

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Sentences and Statements in guide. For teaching suggestions see page 268 - 73 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)	Skillmaster 8 - 62, 63 In Guide: Sentences and Statements open sentences and sentences maintaining Skills

Sentences and Statements

The relation symbols are used to join expressions to form mathematical sentences. Some of the relation symbols are $=$, $>$, $<$, $\geq$, $\leq$, and $\neq$.

An open sentence is a number sentence with a variable.

A sentence that contains enough information to be judged true or false is a statement.

An equation is an open sentence or statement using $=$.

An inequality is an open sentence or statement using $>$, $<$, $\geq$, $\leq$, or $\neq$.

Open sentences		statements	
$n+4=9$	equation	$5+7=10$	false
$n+3>7$	inequality	$7>4$	true
$\frac{3}{5}n=16$	equation	$16+3\neq 19$	false
$-\frac{3}{5}n+4\leq 17$	inequality	$9-3\leq 6$	true

The solution of an open sentence is the number or numbers that replace the variable to make the sentence true. The replacements for the variable are taken from a given set called the domain.

When the domain is $\{0, \frac{1}{5}, 4, -2\}$

if $x+2=6$, solution: 4

if $3x > {}^{-}1$ solution: $0, \frac{1}{5}, 4$

if $3x > 15$ solution: ϕ

Equivalent open sentences have the same solution when their replacements are the same.

Domain: rational numbers

equivalent equations

$x+6=8$

$x+2=4$

$2x-3=1$

$x=2$

solution: 2

Equivalent inequalities

$3y>24$

$2y-5>11$

$5y\geq 40$

$y>8$

solution: $y>8$

Domain: integers

equivalent equations

$x+6=8$ $\quad 2x-3=1$

$x+2=4$ $\quad x=2$

solution: 2

Equivalent inequalities

$3y>24$ $\quad 5y>40$

$2y-5>11$ $\quad y>8$

solution: 9, 10, 11, ...

Identify as an open sentence or a statement:

① $(9+3)-4=7$

② $3(n+6)>15$

③ $6<10$

④ $6n<10$

⑤ $^{-}7+n={}^{-}3$

Given the domain: $^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3$, name the solution.

⑥ $5c>0$

⑦ $n+4>2$

⑧ $2x+{}^{-}3=1$

⑨ $y+4.5=3.5$

Tell which, if any, of the sentences in each group are equivalent. Domain: rational numbers.

⑩ $y+4=8$
$3y+{}^{-}4=6+{}^{-}1$
$y=3$

⑪ $2y<8$
$\frac{1}{2}y<3$
$y<4$

⑫ $3x+2={}^{-}\frac{2}{5}$
$x={}^{-}\frac{4}{5}$
$x+\frac{1}{5}={}^{-}\frac{3}{5}$

⑬ $a=1.4$
$7.3a-2a=4.5$
$a+{}^{-}1.4=2.8$

⑭ $4+x>10.7$
$5+x>12$
$x>6.7$

Sentences and Statements

(Key)

(1) statement

(2) (open) sentence

(3) statement

(4) open sentence

(5) open sentence

(6) 1, 2, 3

(7) −1, 0, 1, 2, 3

(8) 2

(9) −1

(10) $y = 3$
$3y + -4 = 6 + -1$

(11) $2y < 8$
$y < 4$

(12) $x = -4/5$
$3x + 2 = -2/5$
$x + 1/5 = -3/5$

(13) $a = 1.4$
$7.3 - 2a = 4.5$

(14) $x > 6.7$
$4 + x > 10.7$

Maintaining Skills

① $(6\cdot10^3)+(7\cdot10^0)+(6\cdot10^{-3})=$ ____________ (standard numeral)

② $63.4-48.963=$

③ $614.914\approx$ ________ (nearest tens) ________ (nearest units)

④ $\frac{3}{4}=$ ________ %

⑤ $\frac{3}{11}=$ ________ (decimal)

⑥ $63.4\times .09=$ ________

⑦ $693+.94+7.8+53.7=$ ________

⑧ $\frac{6714}{1000}=$ ____________ (decimal)

⑨ $\frac{.38}{10000000}=$ ____________ (decimal)

⑩ $7.046=$ ____________ (fraction)

⑪ $.000007=$ ____________ (fraction)

⑫ $\frac{9}{10}\%=$ ____________ (fraction)

⑬ $.65\%=$ ____________ (fraction)

⑭ $1\frac{3}{7}+2\frac{8}{9}+4\frac{1}{3}=$ ________

⑮ $\frac{4}{9}\cdot\frac{5}{8}\cdot\frac{36}{42}\cdot\frac{16}{15}=$ ________

Maintaining Skills

(Key)

① 6007.006
② 14.437
③ 610 615
④ 75%
⑤ $.\overline{27}$
⑥ 5.706
⑦ 755.44
⑧ 6.714
⑨ .0000038
⑩ $\frac{3523}{500}$
⑪ $\frac{7}{1000000}$
⑫ $\frac{9}{1000}$
⑬ $\frac{13}{2000}$
⑭ $8\frac{41}{63}$
⑮ $\frac{16}{63}$

Mathematics Objective

45 Solve an equation or enequality in the form $x \pm b = c$, where b and c are rational numbers.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Solving equations using addition	Solving equations using addition
For teaching suggestions see p. 88 - 89, 164 - 165 8 (Scott)	For teaching suggestions see p. 100 - 101 7 (Scott)

Addition Property of Equations

$y + 3 = 11$
$y + 3 + {}^{-}3 = 11 + {}^{-}3$
$y + 0 = 8$
$y = 8$

check: $y + 3 = 11$
$8 + 3 = 11$
$11 = 11$

$3.2 - 7.6 = n + 2.13$
$3.2 + {}^{-}7.6 = n + 2.13$
${}^{-}4.4 = n + 2.13$
${}^{-}2.13 + {}^{-}4.4 = n + 2.13 + {}^{-}2.13$
${}^{-}6.53 = n + 0$
${}^{-}6.53 = n$

check: $3.2 - 7.6 = n + 2.13$
$3.2 + {}^{-}7.6 = {}^{-}6.53 + 2.13$
${}^{-}4.4 = {}^{-}4.4$

Show all steps to find solution. check.

① $m + {}^{-}11.5 = {}^{-}2.7 + 0.4$

② $x + ({}^{-}0.3 + {}^{-}1.1) = 43.7$

③ $y + {}^{-}\frac{1}{5} = {}^{-}\frac{3}{10} + \frac{1}{100}$

④ $x + {}^{-}({}^{-}74.37 + 1.12) = {}^{-}0.011$

⑤ ${}^{-}\frac{7}{16} + ({}^{-}\frac{3}{4}) = ({}^{-}\frac{1}{8} - \frac{1}{4}) + m$

⑥ $x + \frac{2}{3} = 7 + {}^{-}\frac{1}{6}$

⑦ $m - {}^{-}3 = 17 + {}^{-}5$

⑧ $m + {}^{-}\frac{2}{3} = \frac{7}{8}$

⑨ $4 + m = {}^{-}10$

⑩ $16.3 - 9 = {}^{-}7.14 + m$

⑪ $x - 1.5 = 3.5$

⑫ $m + \frac{3}{4} = \frac{2}{3}$

Addition Property of Inequalities

$y - 1 > 10$

$y + {}^{-}1 + 1 > 10 + 1$

$y > 11$

check:

let $y = 12$

$y - 1 > 10$

$12 - 1 > 10$

$11 > 10$

$y + \frac{1}{2} \geq 3$

$y + \frac{1}{2} + {}^{-}\frac{1}{2} \geq 3 + {}^{-}\frac{1}{2}$

$y \geq 2\frac{1}{2}$

check: let $y = 3$

$y + \frac{1}{2} \geq 3$

$3 + \frac{1}{2} \geq 3$

$3\frac{1}{2} \geq 3$

$-1 < y - 4 < 10$

$-1 < y + {}^{-}4 < 10$

$-1 + 4 < y + {}^{-}4 + 4 < 10 + 4$

$3 < y + 0 < 14$

$3 < y < 14$

check: let $y = 4$ let $y = 13$

$-1 < 4 - 4 < 10$ $-1 < 13 - 4 < 10$

$-1 < 0 < 10$ $-1 < 9 < 10$

show all steps to find the solution. show check.

① $1 + n \geq {}^{-}9$

② $y + {}^{-}4 < {}^{-}7$

③ $\frac{1}{2}n + \frac{1}{2}n \geq {}^{-}\frac{3}{4} + \frac{1}{8}$

④ $x - \frac{7}{8} \leq \frac{2}{3} + \frac{3}{5}$

⑤ $.6y + .4y \leq {}^{-}\frac{1}{16} + {}^{-}\frac{5}{8}$

⑥ ${}^{-}2 < n - 7 \leq {}^{-}7 + 20$

⑦ $n + 3 \geq {}^{-}6$

⑧ $6.4 + n \geq 2.8$

⑨ $0 < y + 3 \leq 12$

⑩ ${}^{-}2 \leq y - 7 < 9$

Practice

Rational number operations: simplify.

① $(^-\frac{1}{4} - .36) + -(.16 + ^-.3) =$

② $\left(\frac{^-1.6 + ^-.8}{-4}\right) - \left(\frac{4.7 + ^-.3}{11}\right) =$

③ $\left(1\frac{4}{5} \div {^-4}\frac{1}{3}\right) + \left(2\frac{1}{2} \cdot \frac{3}{4}\right) =$

④ $-\frac{3}{4} \cdot \frac{5}{2} \cdot \frac{-16}{9} \cdot \frac{8}{25} \cdot {^-}\frac{30}{32} =$

Addition property: Find solution, check.

⑤ $w - (17.1 + ^-.94) = 24.3 - 16$

⑥ $y + (^-16 - ^-30) = 34$

Addition Property of Equations

① $n = 9.2$
② $z = 45.1$
③ $y = -\frac{9}{100}$
④ $z = -73.261$
⑤ $n = -\frac{13}{16}$
⑥ $z = 6\frac{1}{6}$
⑦ $n = 9$
⑧ $n = 1\frac{13}{24}$
⑨ $n = -14$
⑩ $n = 14.44$
⑪ $x = 5$
⑫ $n = -\frac{1}{12}$

Addition Property of Inequalities

① $n \geq -10$
② $y < -3$
③ $n \geq -\frac{5}{8}$
④ $z \leq 2\frac{17}{120}$
⑤ $y \leq -\frac{11}{16}$
⑥ $5 < n < 20$
⑦ $n \geq$ [illegible]
⑧ $n \geq -3.6$
⑨ $-3 < y < 9$
⑩ $5 \leq y < 16$

Practice

① $-.47$
② $.2$
③ $1\frac{239}{520}$
④ -1
⑤ $n = 24.46$
⑥ $y \geq 20$

Mathematics Objective

46 Solve an equation in the form $a x = b$ where a and b are rational numbers

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Equations in the form $ax = b$ For teaching suggestions see p. 100 - 101., 150 - 154 8 (Scott ____)	Equations in the form $ax = b$ For teaching suggestions see p. 54 - 57, 120 - 121 7(Scott) practice 8 - 46, 48
Use reciprocals to fine equivalent sentences Tor teaching suggestions see p. 279 - 283 8(Silver B.)	Skillmaster 8 - 65
	In Guide: Multiplication Property of equations. Properties of equations and inqualities - (practice) answers Inequalities with two relation symbols.
	MRC: Key to algebra Book 3 AWA II p. 20 PAWP - DD - 47, 48

Multiplication Property of Equations and Inequalities

$3n = 24$
$\frac{1}{3} \cdot 3n = 24 \cdot \frac{1}{3}$
$1 \cdot n = 8$
$n = 8$

check: $3n = 24$
$3 \cdot 8 = 24$
$24 = 24$

$\frac{2}{3}x < \frac{1}{4}$
$\frac{3}{2} \cdot \frac{2}{3}x < \frac{1}{4} \cdot \frac{3}{2}$
$1 \cdot x < \frac{3}{8}$
$x < \frac{3}{8}$

check: $\frac{1}{4} < \frac{3}{8}$
$\frac{2}{3}x < \frac{1}{4}$
$\frac{2}{3} \cdot \frac{1}{4} < \frac{1}{4}$
$\frac{1}{6} < \frac{1}{4}$

① $\frac{1}{8}y = 5$

② $-\frac{2}{7}x = {}^{-}6$

③ $2\frac{2}{3}y = 120$

④ $\frac{1}{4}y \leq 7 + {}^{-}6 + 18$

⑤ $7y > \frac{4}{9}$

Inequalities using a negative multiplier

$-3n > 7$
$-\frac{1}{3} \cdot {}^{-}3n < {}^{-}\frac{1}{3} \cdot 7$
$n < -2\frac{1}{3}$

check: $-3 < -2\frac{1}{3}$
$-3n > 7$
$-3 \cdot {}^{-}3 > 7$
$9 > 7$

⑥ $-4y \geq 37 - 20$

⑦ $-5n + \left(\frac{1}{5} - \frac{2}{3}\right) \geq \frac{14}{15}$

Answers

Multiplication property of equations and inequalities.

① 40

② 21

③ 50

④ $y \le 76$

⑤ $y > \frac{4}{63}$

⑥ $y \le {}^{-}4\frac{1}{4}$

⑦ $n \le {}^{-}\frac{7}{25}$

Properties of equations and inequalities

① $x = -16$

② $w = 1\frac{1}{4}$

③ $y = 15$

④ $x = 9$

⑤ $a \le 2$

⑥ $x > 2$

⑦ $-5 \le n < 7$

⑧ $n > {}^{-}7.53$

⑨ $x = 2$

⑩ $x > {}^{-}54$

⑪ $n \ge 11\frac{2}{3}$

⑫ $n \ge -9$

Inequalities with two relation symbols.

③ ${}^{-}3 < n < 3\frac{1}{4}$

④ $2 < n < 23$

⑤ $-1\frac{1}{4} < n < 23\frac{3}{4}$

⑥ $\frac{1}{5} < n < 3\frac{1}{5}$

Properties of Equations and Inequalities
(practice)

show all steps.

① $3x = -48$

② $-w + \frac{1}{2} = -\frac{3}{4}$

③ $-\frac{1}{5}y = -3$

④ $7\frac{4}{9}x + 1\frac{1}{3}x = 79$

⑤ $\frac{1}{6}a \leq \frac{1}{3}$

⑥ $-2x + 5 < 1$

⑦ $^-2 \leq n + 3 < 10$

⑧ $n + 3.4 > 4.17 - 9.3$

⑨ $\frac{4}{9}x = \frac{1}{3} + \frac{5}{9}$

⑩ $\frac{^-5}{6}x < 45$

⑪ $18n - 15n \geq 35$

⑫ $4n \geq -36$

Inequalities with two relation symbols

Find the solution. Domain: rational numbers

① $2 < x+10 < 5$

$2+{}^-10 < x+10+{}^-10 < 5+{}^-10$

$^-8 < x+0 < {}^-5$

$^-8 < x < {}^-5$

② $13 < 3y-2 < 24$

$2+13 < 3y+{}^-2+2 < 24+2$

$15 < 3y < 26$

$\frac{1}{3}\cdot 15 < \frac{1}{3}\cdot 3y < \frac{1}{3}\cdot 26$

$5 < y < 8\frac{2}{3}$

③ $^-10 < 4n+2 < 15$

④ $4-6 < n-4 < 7+12$

⑤ $^-.5+{}^-.7 \leq .4n-.7 \leq 1.6+.9$

⑥ $6 < 5n+5 < 21$

Mathematics Objective

47 Use the distributive property to combine like terms

48 Use addition and multiplication to solve equations and inequalities.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Distributive Property	In Guide: Distributive Property
For teaching suggestions see p. 17 8 (Scott -)	Multiplication and addition Maintaining skills 1, 2
Using addition and multiplication to solve equations and inequalities	
not found in 8 (Scott -)	Skillmaster 8 - p. 67, 68
Distribution Property	
For teaching suggestions see p. 285 8 (Silver -)	MRC: PAWP: AA66 AWA IV: 20, 22, 28 PAWP: DD - 10, 11, 13, 14, 16, 17, 19, 20, 21
Using addition and multiplication to solve equations and inequalities	Key to algebra - Book 3 Book 4 (8 - 13)
For teaching suggesitons see p. 287 - 292	
8 (Silver - Burdett)	

Distributive Property

Complete the following table

	x	y	$x \cdot y$	$2x \cdot y$	$x \cdot 2y$	$2(x+y)$	$2x+2y$
1.	$^-\frac{3}{4}$	$\frac{1}{5}$					
2.	1.3	5.4					
3.	$-\frac{5}{8}$	$-\frac{3}{5}$					
4.	$.5$	$^-1.4$					

Rename each sum as a product and each product as a sum. Do not simplify.

5. $6w + 2w =$
6. $7a + 14a =$
7. $4(a + b) =$
8. $3(5 + x) =$
9. $a(7 + 1) =$
10. $2a(3 + 4) =$
11. $5a + 4a =$
12. $3w + 7w =$

Use the distributive property to combine like terms.

13. $7a + 3a + {}^-4a =$
14. $^-.5x + 4.3x =$
15. $\frac{1}{2}x + \frac{1}{2}x =$
16. $\frac{3}{4}x + {}^-\frac{1}{2}x =$
17. $5a + {}^-2a + {}^-6a =$
18. $^-1.46x + 9x + {}^-6.5x =$

Distributive Property

Complete the following table

	x	y	$x\cdot y$	$2x\cdot y$	$x\cdot 2y$	$2(x+y)$	$2x+2y$
1.	$-\frac{3}{4}$	$\frac{1}{5}$	$-\frac{3}{20}$	$-\frac{3}{10}$	$-\frac{3}{10}$	$-\frac{11}{10}$	$-\frac{11}{10}$
2.	1.3	5.4	7.02	14.04	14.04	13.4	13.4
3.	$-\frac{5}{8}$	$-\frac{3}{5}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$-\frac{49}{20}$	$-\frac{49}{20}$
4.	.5	$^-1.4$	$^-.7$	$^-1.4$	$^-1.4$	$^-1.8$	$^-1.8$

Rename each sum as a product and each product as a sum. Do not simplify.

5. $6n + 2n = n(6+2)$
6. $7a + 14a = 7a(1+2)$
7. $4(a+b) = 4a + 4b$
8. $3(5+x) = 3\cdot 5 + 3\cdot x$
9. $a(7+1) = 7a + 1a$
10. $2a(3+4) = 3\cdot 2a + 4\cdot 2a$
11. $5a + 4a = a(5+4)$
12. $3n + 7n = n(3+7)$

Use the distributive property to combine like terms.

13. $7a + 3a + {}^-4a = a(7 + 3 + {}^-4) = 6a$
14. $^-.5x + 4.3x = x(^-.5 + 4.3) = 3.8x$
15. $\frac{1}{2}x + \frac{1}{2}x = x(\frac{1}{2} + \frac{1}{2}) = x$
16. $\frac{3}{4}x + {}^-\frac{1}{2}x = x(\frac{3}{4} + {}^-\frac{1}{2}) = \frac{1}{4}x$
17. $5a + {}^-2a + {}^-6a = a(5 + {}^-2 + {}^-6) = {}^-3a$
18. $^-1.46x + 9x + {}^-6.5x = x(-1.46 + 9 + -6.5) = 1.04x$

Multiplication and Addition

When both properties are used and the expression containing the variable is a sum, it is usually best to use the addition property first.

$$
\begin{aligned}
{}^{-}\tfrac{1}{3}y + 4 &= 6 \\
{}^{-}\tfrac{1}{3}y + 4 + {}^{-}4 &= 6 + {}^{-}4 \\
{}^{-}\tfrac{1}{3}y &= 2 \\
-\tfrac{3}{1} \cdot {}^{-}\tfrac{1}{3}y &= 2 \cdot {}^{-}3 \\
y &= {}^{-}6
\end{aligned}
$$

check:

$$
\begin{aligned}
{}^{-}\tfrac{1}{3}y + 4 &= 6 \\
{}^{-}\tfrac{1}{3} \cdot {}^{-}6 + 4 &= 6 \\
2 + 4 &= 6 \\
6 &= 6
\end{aligned}
$$

Find the solutions to the following. Show the steps. Check.

① $3n + {}^{-}2n = {}^{-}({}^{-}3 + 1)$

② $\frac{1}{5}n - \frac{4}{5} = {}^{-}\frac{2}{3} + \frac{1}{4}$

③ $\frac{1}{2}y + \frac{3}{7} = \frac{3}{4} \div {}^{-}\frac{2}{7}$

④ $6n + 4n - 2n = 5 + {}^{-}6 + 11$

⑤ $y + \frac{1}{2}y = \frac{9}{10}$

⑥ $4n + {}^{-}2n \geq {}^{-}7 + 13 - 4$

⑦ $\frac{1}{2}n - \frac{1}{3}n + 3 < \frac{4}{9} - \frac{7}{18}$

⑧ $5.6x - 7.45x + .9 = .64 + {}^{-}.8$

Maintaining Skills (1)

Simplify the following:

① $\left(\frac{3}{5}+\frac{7}{8}\right)+\left(^{-}3\frac{1}{4}+{}^{-}5\frac{1}{3}\right)=$

② $\left[\frac{1}{4}+\left(^{-}\frac{2}{3}+{}^{-}\frac{3}{5}\right)+{}^{-}\frac{3}{20}\right]=$

③ $^{-}87.36-{}^{-}33.41=$

④ $^{-}2\frac{3}{4}\div 3\frac{1}{8}=$

⑤ $(^{-}5)(^{-}\frac{4}{3})(^{-}3.3)=$

⑥ $1.407-2.004=$

⑦ $\left(\frac{26}{70}+\frac{1}{35}\right)+{}^{-}\frac{1}{7}=$

⑧ $(71.65-{}^{-}3.42)-({}^{-}.014+{}^{-}.86)=$

⑨ $1.08=$ ______ %

⑩ $\frac{5}{12}=$ ______ %

⑪ $\frac{2}{11}\%=$ _______ (fraction)

⑫ $4\frac{1}{2}\%=$ _______ (fraction

Maintaining Skills (2)

① 15% of what number is 23?

② What percent of 73 is 7?

③ 24 is what percent of 18?

④ 1.6% of 4.2 is what number?

⑤ 16% of what number is 7?

⑥ 21 is what percent of 35?

⑦ 125% of 6 is what number?

⑧ 19% of 36 is what number?

⑨ 16 is 3% of what number?

⑩ 17% of what number is 63?

⑪ $\frac{5}{13}$ = __________ (decimal)

⑫ $\frac{1}{9}$% = __________ (fraction)

⑬ .025 = __________ (fraction)

⑭ .091 = __________%

⑮ $4\frac{1}{3}$% = __________ (fraction)

Multiplication and Addition

(Key)

① $n = 2$
② $n = 1\frac{11}{12}$
③ $y = -6\frac{3}{28}$
④ $n = 1\frac{1}{4}$

⑤ $y = \frac{3}{5}$
⑥ $n \geq 1$
⑦ $n < {}^{-}17\frac{2}{3}$
⑧ $n = \frac{106}{185}$

Maintaining Skills 1

(Key)

① ${}^{-}7\frac{13}{120}$
② $-1\frac{1}{6}$
③ ${}^{-}53.95$
④ $-\frac{22}{25}$
⑤ -22
⑥ ${}^{-}.597$

⑦ $\frac{9}{35}$
⑧ 75.944
⑨ 108%
⑩ $41\frac{2}{3}\%$
⑪ $\frac{1}{550}$
⑫ $\frac{9}{200}$

Maintaining Skills 2

(Key)

① $153\frac{1}{3}$
② $9\frac{43}{73}\%$
③ $133\frac{1}{3}\%$
④ .0672
⑤ $43\frac{3}{4}$
⑥ 60%
⑦ 7.5
⑧ $6\frac{21}{25}$

⑨ $533\frac{1}{3}$
⑩ $370\frac{10}{17}$
⑪ .38461
⑫ $\frac{1}{900}$
⑬ $\frac{1}{40}$
⑭ 9.1%
⑮ $\frac{13}{300}$

Mathematics Objective

49 Solve a sentence having a variable on both sides of the relation symbol.

50 Solve a compound inequality which represents two sentences joined by "and " or" or."

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Variable on both sides of a sentence.	In Guide: Sentences with variables on both sides
none	Compound inequalities
8 (Scott–)	
Compound inequalities	
none	
8 (Scott–)	
Variables on both sides of a sentence	
For teaching suggestions see p. 293 8 (Silver -)	MRC: PAWP DD – 29, 31, 32, 36, 37, 47, 48
Compound inequalities	Key to alegebra - Book 3
For teaching suggestions see p. 289 8 (Silver B)	

sentences with variables on both sides

$2y + 7 = y + 1$

$2y + {}^{-}y + 7 = y + {}^{-}y + 1$ add y

$y + 7 = 1$

$y + 7 + -7 = 1 + {}^{-}7$ add -7

$y = -6$

$4n - 10 = n - {}^{-}3$

$4n + {}^{-}10 = n + 3$

${}^{-}n + 4n + {}^{-}10 = n + 3 + {}^{-}n$

$3n + {}^{-}10 = 3$

$3n + {}^{-}10 + 10 = 3 + 10$

$3n = 13$

$\frac{1}{3} \cdot 3n = 13 \cdot \frac{1}{3}$

$n = 4\frac{1}{3}$

Find the solution:
Domain: rational numbers

① $-(-7-4) + 3n = 2n - (6-10)$

② $-5w + 4 = 3w - 12$

③ $-\frac{5}{6}y - \frac{1}{6} = {}^{-}\frac{2}{6}y + \frac{11}{6}$

④ $.4y \geq 2.4y + 5.8$

Compound Inequalities

We can write the compound sentence "$x > 4$ or $x = 4$" as "$x \geq 4$." Read "$x \geq 4$" as "x is greater than or equal to 4." The "or" sentence represents the union of two conditions. If at least one of the conditions is true, the compound sentence is true.

The compound sentence "$x > 1$ and $x < 5$" can be written as "$1 < x < 5$". Read "$1 < x < 5$" as "x is greater than one and less than 5". The "and" sentence represents the intersection of two conditions. This compound sentence is true only if both conditions are true.

Identify as true or false:

① $7 > 5$ or $4 > 7$

② $1 < 6, 7, 8 < 8$

③ $4 = 3+1$ and $4 > 3$

④ $6 = 6$ or $6 > 4$

⑤ $^-1 < x \leq 7$; $x = 3$

⑥ $0 < 5 \leq 5$

⑦ $16 \geq 14$

⑧ $7 = 4+2$ and $7 < 10$

⑨ $^-19 \leq {}^-20$

sentences with variables on both sides

(Key)

① $n = {}^{-}7$

② $w = 2$

③ $y = {}^{-}4$

④ $y < {}^{-}2.9$

Inequalities with two relation symbols

③ ${}^{-}3 < n < 3\frac{1}{4}$

④ $2 < n < 23$

⑤ ${}^{-}1.25 \leq n \leq 8$

⑥ $\frac{1}{5} < n < 3\frac{1}{5}$

compound Inequalities

① T
② F
③ T
④ T
⑤ T
⑥ T
⑦ T
⑧ F
⑨ F

Problems involving conditions for inequality.

1.) John has 14 model cars, and Bill has 9. Together John and Bill more model cars than Carl. How many model cars can Carl have? y = number of cars Carl can have.

2.) Jane bought 37 post cards. She mailed some of them and then had fewer than 10 post cards left. How many post cards could Jane have mailed?

3.) John bought some bricks to build a fence. He used 19 of the bricks and had fewer than 6 bricks left. How many bricks could John have bought? y = number of bricks John could have bought.

4.) After Jane put 8 new stamps in her collecion, she had more than 70 stamps. How many stamps could Jane have had in her collection before she put in the 8 new ones? y = the number of stamps Jane could have had at first.

5.) Jill had 85 paper clips she gave 17 of the paper clips to Amy. Then Jill had fewer paper clips than Amy. How many paper clips could Amy had had? y = number of paper clips Amy could have had.

6.) Mr. Smith is 58 years old. Mr. Jones is older than Mr. Smith. The difference in their ages is fewer than 20 years. How many years old can Mr. Smith be? y = how many years old Mr. Smith can be.

7.) Joan had 16 phonograph records. She bought some more, but she still had fewer than 45 records. How many records could Joan have bought? y = number of records Joan could have bought.

8.) Kathy made 30 flavors for a party. After the party, she had fewer than 7 flavors left. If each person got one flavor, what is the least number of people that could have been at the party?

9.) Paul had a mathematics assignment. After he worked 6 problems, he still had more than 20 problems to work. What is the least number of problems that could have been in the assignment?

10.) The highest temperature in Houston one summer was 106 degrees. One afternoom that summer, the temperature dropped 28 degrees but it was still higher than 57 degrees. What could the temperature have been before it started to drop?

11.) The Smith family drove from their home to lake Conroe in two days. Lake Conroe is 856 miles from their home. The first day they drove more than 395 miles. How many miles could they have driven the second day?

12.) It took Mr. Jones 3 hours to set out 130 rose bushes. He set out 46 bushes the first hour and at least one plant during each of the remaining two hours. What is the greatest number of plants he could have set out the second hour?

13.) The product of 13 and 17 is less than what numbers?

14.) To what numbers can you add 8 so that each sum is greater than 23?

15.) What is the greatest number from which 5 can be subtracted so that the difference is less than 15?

16.) Simple condition with two variables.

Bob and James went hunting. Together they shot fewer than 9 birds. How many birds could each of the boys have shot.

16.)

x								
y								

17.) Together, Mary and June bought fewer than 8 records. Mary bought more than 3. How many records could each of the girls have bought?

(Use whole numbers.)

M								
J								

Mary could have bought
Mary could have bought
Mary could have bought
Mary could have bought

18.) Fewer than 6 students received a grade of A on a Math examination. More than 2 boys received a grade of A. How many boys and how many girls could have received a grade of A on this examination?

19.) In two days Jeff read a 250-page book. The second day he read fewer than 5 pages. How many pages could he have read on each of the two days?

20.) Bill is older than John. In 6 years, Bill will be 18 years old. How old is Bill now? How old can John be now?

21.) Jack and his brother made fewer than 8 model airplans. If Jack had made 2 more, he would have made the same number as his brother. How many model airplans could each of them have made.

22.) Mary answered all 5 questions on a test. She had more correct answers than incorrect answers. How many correct and how many incorrect answers could she have had?

Use whole numbers

C						
I						

23.) Sally had fewer than 9 quarters and dimes. She had the same number of quarters as dimes. How many coins of each kind could she have had?

Q							
D							

24.) Eight years from now, Amy will be more than 22 years old. Five years ago, she was less than 12 years old. How old can Amy be now?

25.) Sam collects foreign stamps. He has more than 10 of them now. If he had 4 fewer foreign stamps, he would have fewer than 9 in all. How many foreign stamps can Sam have now?

26.) A first number is equal to the quotient of 1096 and 137. When a second number is subtracted from the first number, each difference is greater than 5. What can the number be?

Problems involving conditions for inequality.

1.) John has 14 model cars, and Bill has 9. Together John and Bill more model cars than Carl. How many model cars can Carl have? y = number of cars Carl can have.

$14 + 9 > y$
$23 > y$
$y = 0, 1, 2, \dots 22.$

2.) Jane bought 37 post cards. She mailed some of them and then had fewer than 10 post cards left. How many post cards could Jane have mailed?

y = number of post cards that Margaret could have mailed.

$37 - y < 10$
$y = 28, 29, 30 \dots 37$

3.) John bought some bricks to build a fence. He used 19 of the bricks and had fewer than 6 bricks left. How many bricks could John have bought? y = number of bricks John could have bought.

$y - 18 < 6$
$y = 18, 19, 20 \dots 23$

4.) After Jane put 8 new stamps in her collecion, she had more than 70 stamps. How many stamps could Jane have had in her collection before she put in the 8 new ones? y = the number of stamps Jane could have had at first.

$y + 8 > 70$
$y = 63, 64, 65 \dots$

5.) Jill had 85 paper clips she gave 17 of the paper clips to Amy. Then Jill had fewer paper clips than Amy. How many paper clips could Amy had had? y = number of paper clips Amy could have had.

$85 - 17 < y$
$68 < y$
$y = 69, 70, 71, \dots$

6.) Mr. Smith is 58 years old. Mr. Jones is older than Mr. Smith. The difference in their ages is fewer than 20 years. How many years old can Mr. Smith be? y = how many years old Mr. Smith can be.

$y - 58 < 20$
$y = 77, 76, 75 \ldots$
Mr. Smith can be 77-58

7.) Joan had 16 phonograph records. She bought some more, but she still had fewer than 45 records. How many records could Joan have bought? y = number of records Joan could have bought.

$13 + y < 45$
$y = 1, 2, 3, \ldots 31$

8.) Kathy made 30 flavors for a party. After the party, she had fewer than 7 flavors left. If each person got one flavor, what is the least number of people that could have been at the party?

$30 - y < 7$
24, 25, 26 . . . 30
24

9.) Paul had a mathematics assignment. After he worked 6 problems, he still had more than 20 problems to work. What is the least number of problems that could have been in the assignment?

$y - 6 > 20$
27, 28, 29 . . .
27

10.) The highest temperature in Houston one summer was 106 degrees. One afternoom that summer, the temperature dropped 28 degrees but it was still higher than 57 degrees. What could the temperature have been before it started to drop?

$y - 28 > 57$
86, 87, 88 . . .
86 – 106

11.) The Smith family drove from their home to lake Conroe in two days. Lake Conroe is 856 miles from their home. The first day they drove more than 395 miles. How many miles could they have driven the second day?

$856 - y > 395$
1, 2, 3, . . . 460

12.) It took Mr. Jones 3 hours to set out 130 rose bushes. He set out 46 bushes the first hour and at least one plant during each of the remaining two hours. What is the greatest number of plants he could have set out the second hour?

$46 + y < 130$
1, 2, 3, . . . 83
83

13.) The product of 13 and 17 is less than what numbers?

$13 \times 17 < y$
$221 < y$

y = 2, 2, 2, 223, 224 . . .

14.) To what numbers can you add 8 so that each sum is greater than 23?

$y + 8 > 23$
16, 17, 18 . . .

15.) What is the greatest number from which 5 can be subtracted so that the difference is less than 15?

$y - 5 < 15$
5, 6, 7, . . . 19
19

$y - 5 = 15$
$y = 20$

16.) Simple condition with two variables.

Bob and James went hunting. Together they shot fewer than 9 birds. How many birds could each of the boys have shot.
$x + y < 9$ $U = c \times c$

x	1	2	3	4	5,	6,	7,	8
y	1 2 2 7	1 2 2 6	1 2 2 5	1 2 2 4	1 1 2 3	1 2 2	1	X

Bob could have caught 1 James 1 – 7 etc.

17.) Together, Mary and June bought fewer than 8 records. Mary bought more than 3. How many records could each of the girls have bought?

$M + J < 8$
$J > 3$

(Use whole numbers.)

M	0	1	2	3	4	5	6	7
J	0 0 0 7	0 0 0 6	0 0 0 5	0 0 0 4	0 0 0 3	0 0 0 2	0 0 0 1	0 0 0 0

Mary could have bought 0 and June 4 – 7
Mary could have bought 1 and June 4 – 6
Mary could have bought 2 and June 4 – 5
Mary could have bought 3 and June 4

18.) Fewer than 6 students received a grade of A on a Math examination. More than 2 boys received a grade of A. How many boys and how many girls could have received a grade of A on this examination?

$B + G < 6$
$B > 2$

19.) In two days Jeff read a 250-page book. The second day he read fewer than 5 pages. How many pages could he have read on each of the two days?

$x + y = 250$
$y < 5$
$x = 249, 248, 247, 246$
$y = 1, 2, 3, 4$

20.) Bill is older than John. In 6 years, Bill will be 18 years old. How old is Bill now? How old can John be now?

$B > J$
$B + 6 = 18$
$B = 12$
$J = 1, 2, 3, \ldots 11$

21.) Jack and his brother made fewer than 8 model airplans. If Jack had made 2 more, he would have made the same number as his brother. How many model airplans could each of them have made.

$J + B < 8$
$J + 2 = B$

$J + J + 2 < 8$
$2J + 2 < 8$
$J < 3$

Jack 1 Brother 3
Jack 2 Brother 4

22.) Mary answered all 5 questions on a test. She had more correct answers than incorrect answers. How many correct and how many incorrect answers could she have had?

$C + I = 5$
$C > I$

Use whole numbers

C	0	1	2	3	4	5
I	5	4	3	2	1	0

3 correct 2 incorrect
4 correct 1 incorrect
5 correct 0 incorrect

23.) Sally had fewer than 9 quarters and dimes. She had the same number of quarters as dimes. How many coins of each kind could she have had?

$Q + D < 9$
$Q = D$

Q	1	2	3	4	5	6	7
D	1	1	1	1	1	1	1
	2	2	2	2	2	2	
	3	3	3	3	3		
	4	4	4	4			
	5	5	5				
	6	6					
	7						

1 quarter,	1 dime
2 quarter	2 dime
3 quarter	3 dime
4 quarter	4 dime

24.) Eight years from now, Amy will be more than 22 years old. Five years ago, she was less than 12 years old. How old can Amy be now?

$x + 8 > 22$
15, 16, 17, . . .

$x - 5 < 12$
16, 15, 14 . . .

15, 16

25.) Sam collects foreign stamps. He has more than 10 of them now. If he had 4 fewer foreign stamps, he would have fewer than 9 in all. How many foreign stamps can Sam have now?

$x > 10$
11, 12, 13, . . .

$x - 4 < 9$
12, 11, 10 . . .

11, 12

26.) A first number is equal to the quotient of 1096 and 137. When a second number is subtracted from the first number, each difference is greater than 5. What can the number be?

$1096 \div 137 = x$
$x = 8$

$x - y > 5$
$8 - y > 5$
$y = 0, 1, 2$

Mathematics Objective

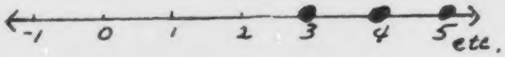
51. Graph solutions of equations and inequalities on a number line.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Graphing on a number line not in 8 (Scott Foresman)	In Guide: Graphing on a number line Graphing solutions
Graph solutions of an inequality p. 290 8 (Silver Burdett)	Multiplication Additional problems (ans)
Word problems p. 291-292 8 (Silver Burdett)	MRC. PAWP BB 27, 28 DD 46, 47, 48

Graphing on a number line

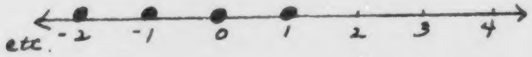
Domain: integers

$x > 2$



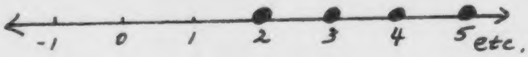
-1
0
1
2
3
4
5
etc.

$x < 2$



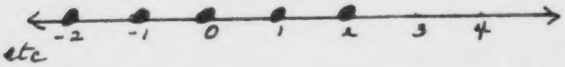
etc.
-2
-1
0
1
2
3
4

$x \geq 2$



-1
0
1
2
3
4
5
etc.

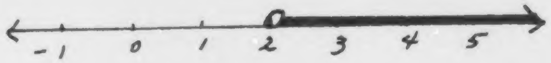
$x \leq 2$



etc
-2
-1
0
1
2
3
4

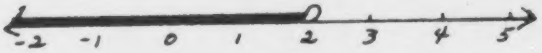
Domain: real numbers

$x > 2$

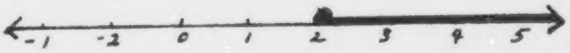


open circle to show that 2 is not in the solution

$x < 2$

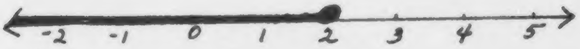


$x \geq 2$

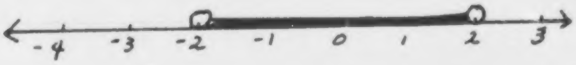


closed circle to show that 2 is a part of the solution.

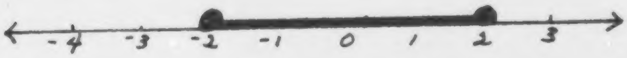
$x \leq 2$



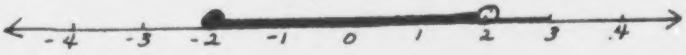
$-2 < x < 2$



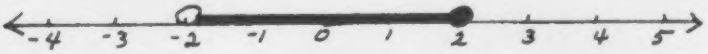
$-2 \leq x \leq 2$



$-2 \leq x < 2$



$-2 < x \leq 2$



-4
-3
-2
-1
0
1
2
3
4
5

Graphing solutions

Draw a number line for each of the following and graph
Domain: real numbers

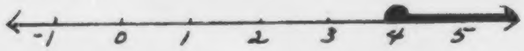
1. $n \geq 4$
2. $6 < x < 10$
3. $x < {}^{-}3$
4. $x = 3$
5. $1 \leq n < 4$
6. ${}^{-}2 \leq n \leq 3$
7. $0 \leq n \leq 4$
8. $y > 5$
9. $x \leq -4$
10. ${}^{-}5 \leq n < {}^{-}1$
11. ${}^{-}3 < n \leq 0$

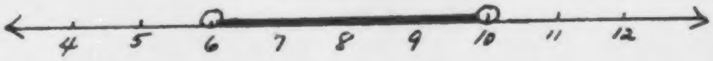
Find the solution set and graph. Domain: real numbers

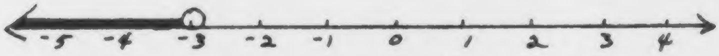
1. $3n + 2n + 6n = 22$
2. $4(2 + n) \geq 16$
3. $3(n + 4) < 17$
4. $2(n + 1) \leq 5$
5. $5(3 + n) > 10$
6. $2(2n - 3) > {}^{-}2$
7. $n + {}^{-}2n + 3n < 4$
8. $6n + 2n - 5n > 9$
9. $-2(n - 3) \leq 8$
10. ${}^{-}3n + 5n - 4n \geq 6$

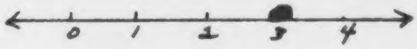
Graphing solutions

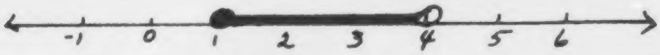
(Key)

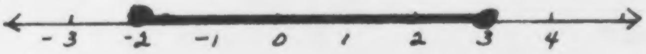
1. 

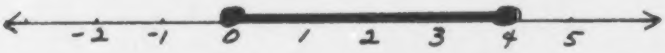
2. 
4
5
6
7
8
9
10
11
12

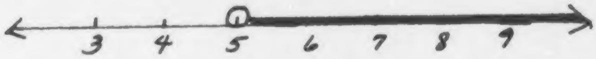
3. 
-5
-4
-3
-2
-1
0
1
2
3
4

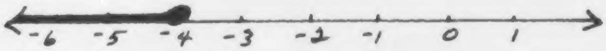
4. 

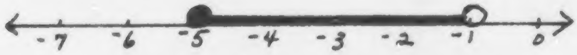
5. 

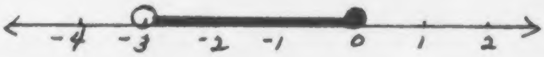
6. 

7. 

8. 

9. 

10. 

11. 

1. $n = 2$
2. $n \geq 2$
3. $n < 1\frac{2}{3}$
4. $n \leq 1\frac{1}{2}$
5. $n > {}^{-}1$
6. $n > 1$
7. $n < 2$
8. $n > 3$
9. $n \geq {}^{-}1$
10. $n \leq {}^{-}3$

Multiplication

1. $3+5\cdot 2=$
2. $5+3(6+4)=$
3. $5\cdot 6-2$
4. $6+3\cdot 5+8-2\cdot 3+6-4-1=$
5. $3[(1+7)\cdot 2]=$
6. $\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{6}{8}=$
7. $4[2\ 3-(4+6)+2]=$
8. $7(x+y)=$
9. $17+3(5+2)=$
10. $22+3-24\div 8=$
11. $12\div 1\cdot 2=$
12. $4(9)-4(1)=$
13. $72(-\frac{1}{4})(-\frac{1}{6})=$
14. $5+(-\frac{1}{2})[8+40(-\frac{1}{5}+\frac{4}{5})]$
15. $3[(5+1)\cdot(6\div 2)]$

Place parenthesis to make statement true

1. $2+3\cdot 7=35$
2. $8-3+4=1$
3. $4+6\cdot 8-3=50$
4. $3\cdot 7+2\cdot 5=115$
5. $9-2\cdot 4-3=4$

Multiplication - Key

1. 13
2. 35
3. 28
4. 24
5. 48
6. $\frac{5}{12}$
7. 60
8. $7x + 7y$
9. 38
10. 22
11. 24
12. 32
13. 3
14. 21
15. 54

1. $(2+3)7 = 35$
2. $8 - (3+4) = 1$
3. $(4+6)(8-3) = 50$
4. $(3 \cdot 7 + 2)5 = 115$
5. $9 - (2 \cdot 4 - 3) = 4$

Additional Problems

Find the solution: Graph all inequalities:

① $\frac{1}{4}y \le 2$

② $y - 6 > 4$

③ $\frac{5}{2}n - \frac{7}{8} = 1$

④ $^{-}a + 7a = 84$

⑤ $8n - 41 = 1 + 2n$

⑥ $8n - 5 \le 4n + 3$

⑦ $6.4 - .4n \ge 2.8$

⑧ $-\frac{4}{5}x > \frac{3}{10}$

⑨ $12 \le 2n + 6 < 18$

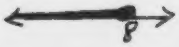
⑩ $n - 17 = 4n - 3$

⑪ $n + {}^{-}7 = 3n - 4 + 7$

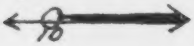
⑫ $3n + 12 \ge -6$

Additional Problems
(Answers)

① $y \le 8$



② $y > 10$

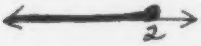


③ $n = \frac{3}{4}$

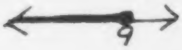
○ $a = 14$

⑤ $n = 7$

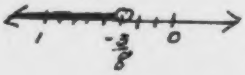
⑥ $n \le 2$



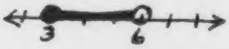
⑦ $n \le 9$



⑧ $n < -\frac{3}{8}$



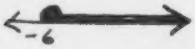
⑨ $3 \le n < 6$



⑩ $n = -4\frac{2}{3}$

⑪ $n = {}^-5$

⑫ $n \ge -6$



Mathematics Objective

52. Use an ordered pair to name and locate points in the coordinate plane

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 180-181 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman) For teaching suggestions see page 382-384 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)	8-L 39, 40 Skillmaster 8-79 In Guide: Getting the Point! MRC: PWAP: DD 54-56 DD 63, 64

ACTIVITIES

Edited by
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Montclair State College, Upper Montclair, New Jersey

GETTING THE POINT!

By Marcia F. Snyder, Menlo Park, CA 94025

Teacher's Guide

Grade level: 7–12

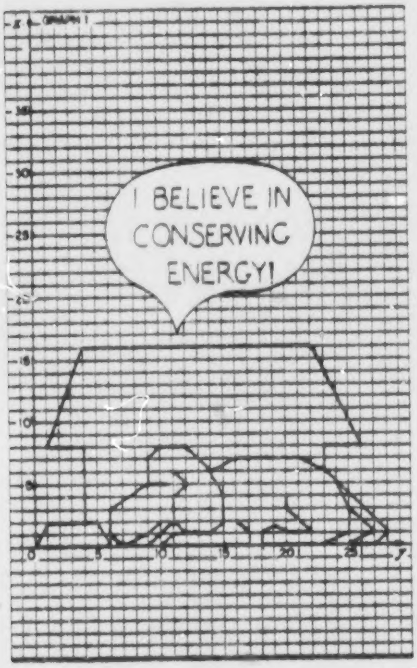
Materials: Worksheet with the listing of points to plot. Graph sheets 1 and 2 with axes drawn. Ruler.

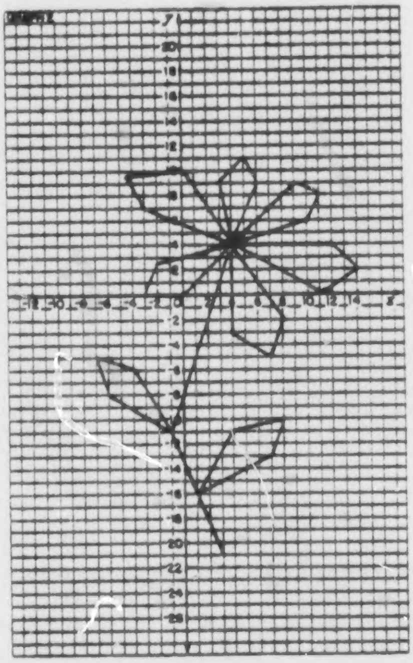
Objectives: Just plotting random points can be boring. These two activities, however, with the promise of turning out dot-to-dot pictures, will generate enthusiasm for learning how to plot points properly.

Graph 1 is a good initial experience in familiarizing students with the rectangular coordinate system by locating points in the first quadrant only.

Graph 2 provides more practice in plotting and emphasizes all four quadrants.

Activity Outcome: Once students have mastered the art of point plotting, why not encourage them to create their own dot-to-dot designs. Give them extra credit for their efforts, and allow other students to work them out, too.





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WORKSHEET: GRAPHING PICTURES

Plot the following points, connecting them one by one. When you finish, you should have a picture.

GRAPH 1

Start at (4, 2)
(5, 2)
(6, 0)
(0, 0)
(1, 2)
(4, 2)
(4, 8)
(1, 8)
(4, 16)
(22, 16)
(26, 8)
(23, 8)
(23, 6)
(21, 7)
(16, 7)
(14, 6)
(12, 8)
(10, 8)
(9, 7)
(9, 5)
(6, 3)
(6, 1)
(7, 0)
(9, 0)
(11, 2)
(12, 1)
(11, 1)
(10, 0)
(17, 0)
(17, 1)
(16, 2) Break

Begin
again at (9, 5)
(10½, 5) Break

Begin
again at (20, 4)
(20, 3)
(22, 1)
(20, 1)
(19, 1½)
(18, 1)
(18, 0)
(23, 0)
(25, 1)
(25, 3)
(24, 5)
(23, 6) Break

Begin
again at (24, 5)
(28, 1)
(27, 0)
(25, 0)
(27, 1)
(25, 3) Break

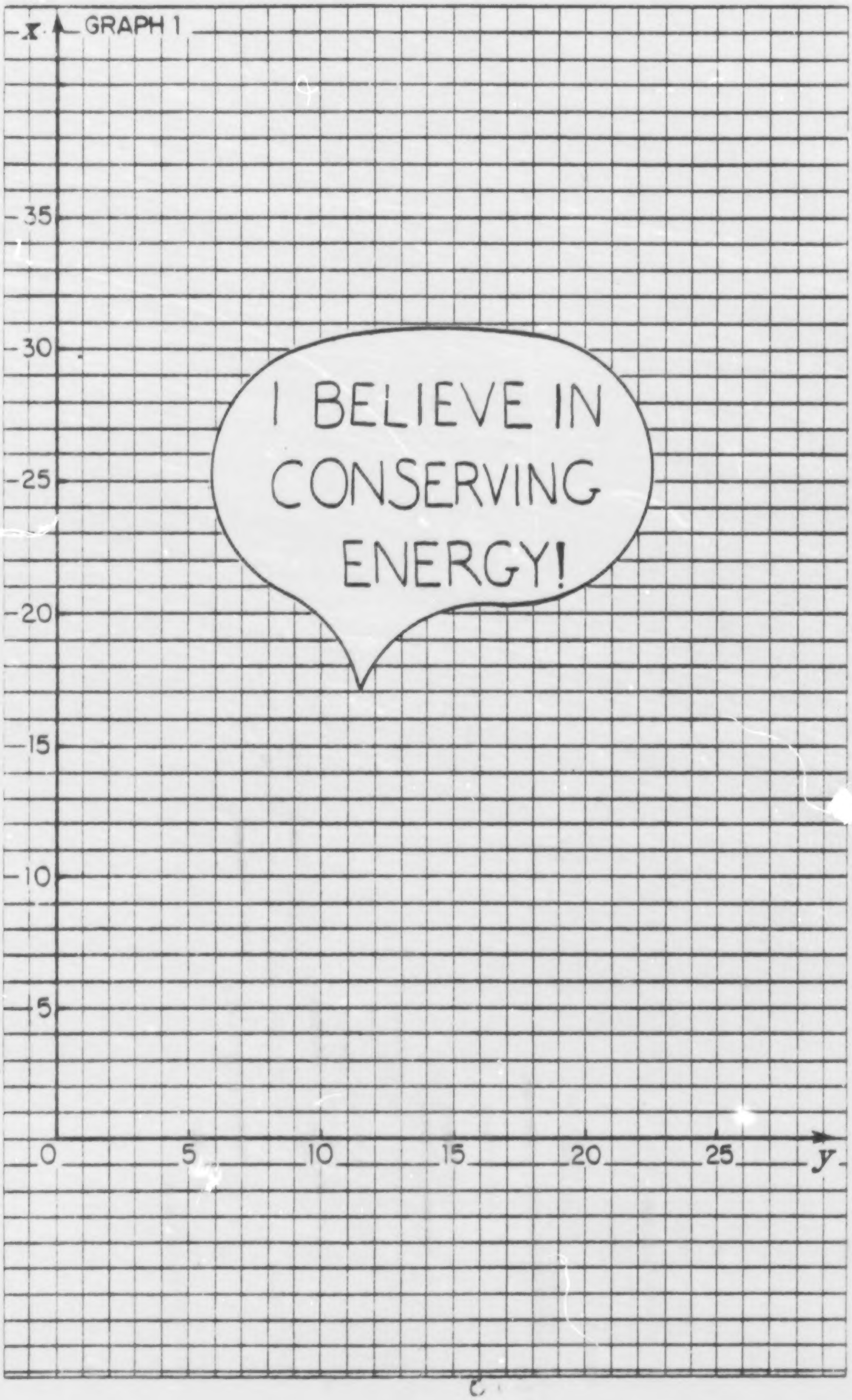
Begin
again at (11, 6)
(12, 5)
(11, 4)
(11, 2)
(12, 1)
(14, 1)
(15, 2)
(15, 4)
(14, 6) Break

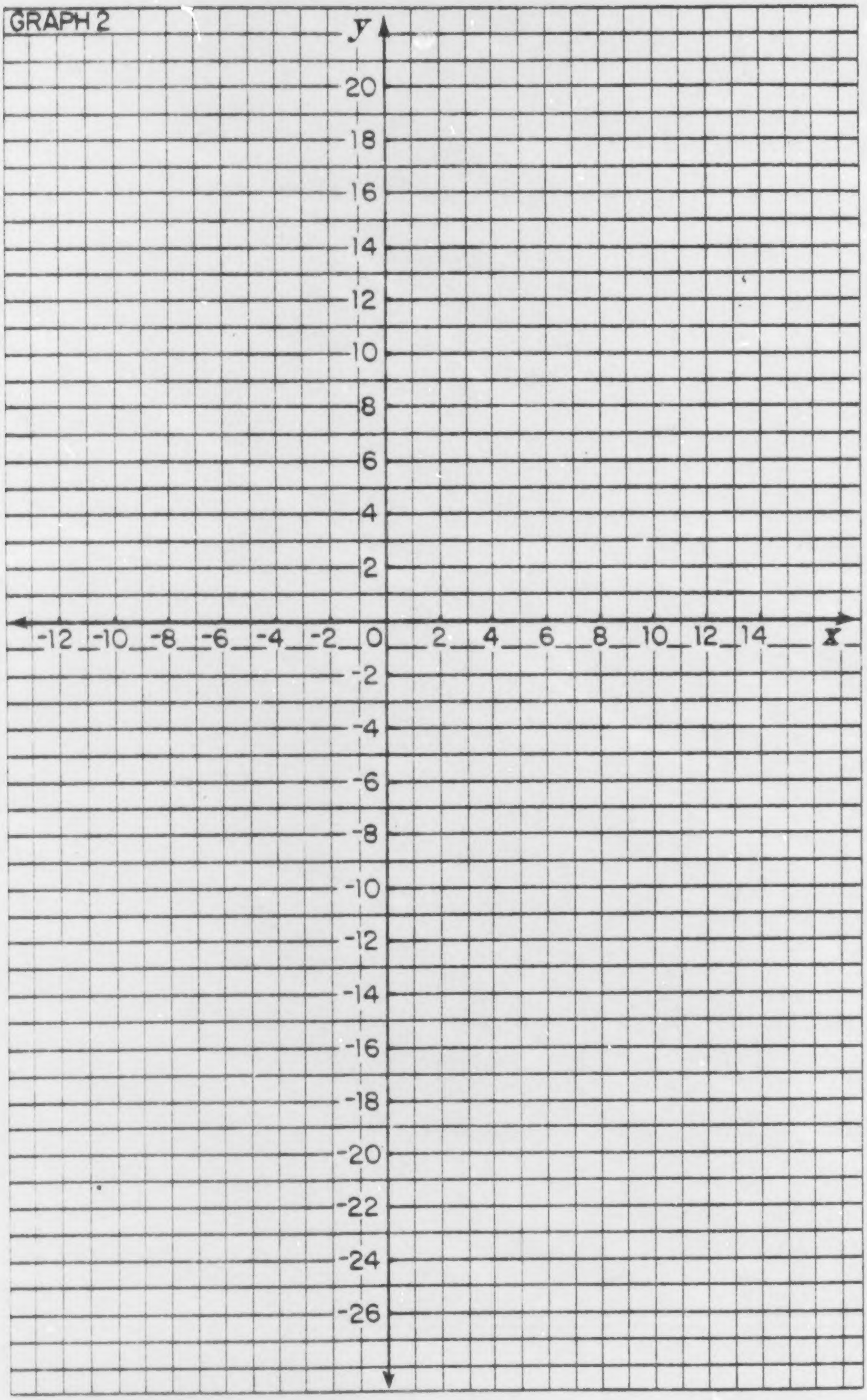
Begin
again at (7, 0)
(9, 1)
(10, 2) Stop

GRAPH 2

Begin at (4, 4)
(0, 0)
(-3, 0)
(-2, 2½)
(4, 4)
(-2, 7)
(-3½, 9½)
(0, 10)
(4, 4)
(3, 9)
(5, 11)
(6, 9)
(4, 4)
(9, 9)
(11, 8)
(10, 6)
(4, 4)
(12, 4)
(14, 2)
(11, 0)
(4, 4)
(8, -2)
(7, -5)
(4, -3)
(4, 4)
(-1, -11)
(-4, -6)
(-7, -5)
(-6, -8)
(-1, -11)
(3, -21) Break

Begin
again at (1, -16)
(4, -11)
(8, -10)
(7, -13)
(1, -16) Stop


GRAPH 1
x
35
30
25
20
15
10
5
0
5
10
15
20
25
y
I BELIEVE IN CONSERVING ENERGY!


GRAPH 2
y
20
18
16
14
12
10
8
6
4
2
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
x
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
-26

Mathematics Objective

53. Graph the equation of lines by plotting points on the line and determine whether or not a given point lies on a line.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Reading and making line graphs For teaching suggestions see page 172, 176-7 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)	
Making line graphs for equations in four quadrants For teaching suggestions see page 182-183 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)	8-H19, 20 practice 8-76, 77, 78 Teacher aid 5
Finding ordered pairs for relations defined by inequalities For teaching suggestions see page 386, 388 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)	MRC: PWAP: DD-54 DD-55 DD-56 DD-63, 64 DD-57, 58
Finding ordered pairs for relations defined by equations For teaching suggestions see page 389-390 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)	
Make line graphs for equations For teaching suggestions see page 394-95 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)	

Mathematics Objective

53a Use graphs to solve problems
53b Solve motion problems

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 184-187 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) For teaching suggestions see page 398 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)	

Mathematics Objective

68. read and interpret graphs and charts

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Graphs For teaching suggestions see page 302, 304, 306, 308 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) Line graph and bar graph S.B. p. 350-52 Collect examples of circle graphs and discuss these with the students. flow charts: S.B. p 183, 211, p. 239, p. 392	Graphs p. 348-352 7 (Scott Foresman) practice 8-109, 110, 111, 112 8-H 32 Graphs p. 356-361 7 (Silver Burdett) Skillmaster 8-76, 77 In Guide: Maintaining Skills

Mathematics Objective

69. make and interpret a frequency table.
70. find the mean, median, or mode of a set of data.

Teaching Techniques and Introductory Activities

frequency chart - collecting data

For teaching suggestions see page 360-362 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Mean, Median, Mode, Range

For teaching suggestions see page 366-68 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

frequency chart

SB. p. 353-5

Mean, median, mode, range

p. 357, 358

Collecting Data
p. 344-351

Supplementary Activities and Resources

Collecting Data
p. 344-351
7 (Scott Foresman)

Mean, Median, Mode, Range
p. 352-360
7 (Scott Foresman)

practice 8-125, 126, 127, 128
8-L 60
8-H 36

data - frequency diagram
p. 336-341
7 (Silver Burdett)

Mean, Median, Mode, Range
p. 342-365
7 (Silver Burdett)

In Guide: How Many Ways
Word Search
Maintaining Skills

Maintaining Skills

1. $$\frac{\left(\frac{3}{4} + \frac{1}{3}\right)}{4} \div \frac{\left(\frac{2}{3} \cdot \frac{1}{5}\right)}{5} =$$

2. (6.3) (5.2) (90.1) (0) (4) =

3. $^{2}/_{3}$ % = ________________ (fraction)

4. .056% = ________________ (decimal)

5. 70 = ________________ (percent)

6. $^{4}/_{9}$ = ________________ (percent)

7. Base salary = ________________
   total salary = \$1200
   sales = \$5000
   rate = 5%

8. Base salary = \$400
   sales = \$7200
   rate of commission = 4%
   commission = ________________
   Total salary = ________________

9. What percent of 24 is 13? ________________

10. 23 is what percent of 66? ________________

Answers:

1. 10 $^{5}/_{32}$
2. 0
3. $^{1}/_{150}$
4. .00056
5. 7000%
6. 44 $^{4}/_{9}$ %
7. \$950
8. \$288
   \$688
9. 54 $^{1}/_{6}$ %
10. 34 $^{28}/_{33}$ %

Mathematics Objective

67. Solve various types of word problems

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
	MRC: PWAP DD 40, 41, 42, 43 In Guide: Solving Problems (answers)

Solving Problems

1. Identify the unknown
2. Translate each problem into an open sentence
3. Solve the open sentence
4. Check that your solution is a reasonable answer to the problem.

① The sum of two whole numbers is 106. One number is two less than twice the other number. What are the numbers?

② Two sides of a triangle that have the same length are each 5 inches longer than the third side. If the perimeter is 31 inches, how long is each side?

③ Leila is 7 years older than Pam. Their combined age is 37 years. How old is each girl?

④ The length of a rectangle is 8 inches more than its width. The perimeter of the rectangle is 120 inches. What are the dimensions of the rectangle?

⑤ Eight more than twice a certain number is 50. What is the number?

⑥ The length of a rectangular region is twice the width. The area of the rectangular region is 450 cm^2. What are the length and width?

⑦ Joe is twice as old as Jerry and Jerry is three times as old as Don. If their combined age is 40 years, how old is each boy?

⑧ The length of a rectangle is 2 inches less than 3 times its width. If the perimeter of the rectangle is 44 inches, what is its length and width?

⑨ The area of a rectangle is 54 in.^{2} and the width is 6 in. What is the length?

⑩ The width of a rectangle is 10 cm and the perimeter is 50 cm. What is the length?

⑪ Matthew has 12 coins consisting of nickels and dimes. If he has 4 more nickels than dimes, how much money does he have?

⑫ Emilie has 7 more dimes than he has nickels. If the value of his coins is $2.20, how many of each coin does he have?

⑬ In 3 years Judy will be three times as old as Gwen. Judy is now 10 years older than Gwen. How old is Gwen now?

⑭ Sam is 5 years older than Mike. In 6 years the sum of their ages will be 25. How old is each boy now?

⑮ The sum of three consecutive integers is 105. What are the integers?

⑯ Find two consecutive even integers whose sum is 46.

⑰ The sum of two numbers is 55. The smaller is equal to 4 more than half the larger. What are the numbers?

⑱ The larger of two numbers is four times the smaller. Their difference is 63. What are the two numbers?

⑲ The ratio of two numbers is equal to 2:7, and the difference between the two numbers is 30. What is the smaller of the two numbers?

⑳ A number is one-third of another number and their sum is -36. what are the numbers?

(21) In a high school, the ratio of the number of boys to the number of girls is equal to 9:7. If there are 1120 pupils in the school, what is the number of boys? the number of girls?

(22) If 17 is added to three times a number, the result is the same as it is when 5 is subtracted from five times the number. What is the number?

(23) John has $48 and Pete has $9. John gives some of his money to Pete. After Pete receives the money, John then has only twice as much as Pete. How much money did John give to Pete?

(24) Janet went to the bank with a twenty-dollar bill and asked for change. She wanted the money in nickels, dimes, and quarters, and the same number of each kind of coin. How many of each did she receive?

Solving Problems
(Answers)

① 36, 70
② 7, 12
③ 15, 22
④ 26, 34
⑤ 21
⑥ $l = 15$ cm
$w = 30$ cm
⑦ Joe is 24 yrs. old
Jerry is 12 yrs old
Don is 4 yrs old
⑧ $w = 6$ in.
$l = 16$ in.
⑨ 9 in.
⑩ 15 cm
⑪ 80¢
⑫ 10 nickels
17 dimes
⑬ 2 yrs old
⑭ Mike is 4 yrs old
Sam is 9 yrs old
⑮ 34, 35, 36
⑯ 22, 24
⑰ 31, 34
⑱ 21 and 84
⑲ 12
⑳ $^{-}9$ and $^{-}27$
㉑ 630, 490
㉒ 11
㉓ #10
㉔ 50

Mathematics Objective

71. decide when to estimate or give an exact count in a given situation.
71a. find total number of choices by making a tree diagram or using the counting principle.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Counting: Guessing and Estimating 8 (Scott Foresman)	Predicting Outcomes p. 235-339 7 (Scott Foresman)
Tree diagram For teaching suggestions see page 236-238, of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)	Practice 8-113, 114, 115, 116, 117 8-L 56, 57
SB p. 366, 367, 371	Possible outcomes p. 402-408 7 (Silver Burdett)

Mathematics Objective

72. Find the probability when the choices are equally likely
72a. Find the probability when the choices are independent or dependent.
72b. Make predictions by multiplying probabilities or by adding probabilities.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 342, 348, 354, 356 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman) Probability of an event p. 373-380 8 (Silver Burdett)	practice 8, 119, 120, 121, 122, 123, 124 8 L 59 8 H 35 Probability of events p. 409-413 7 (Silver Burdett) MRC: PWAP: BB 57, 61, 62

Probability

A box contains 3 marbles, 1 red, 1 black, 1 white. Three cards marked 1, 2, and 3 are also in the box. All possible outcomes or choices can be found by making a table, a tree diagram, or by using the counting principle.

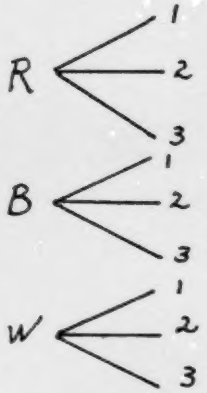
Table

card	R	B	W
3	R 3	B 3	W 3
2	R 2	B 2	W 2
1	R 1	B 1	W 1

Marble

9 choices

Tree Diagram



9 choices

Counting Principle

Number of Marbles X Number of cards

3 X 3 = 9

9 choices

The probability of each choice or event is $\frac{1}{9}$.

The probability of drawing a red marble is $\frac{3}{9}$ or $\frac{1}{3}$.

If each draw is replaced in the box before a second draw, the choices are independent. They have no influence on each other.

If each draw is not replaced in the box before a second draw, the choices are dependent. Suppose a red marble is picked with a card on the first draw and is not replaced, the probability of getting a red marble on the second draw will be $\frac{2}{8}$.

① How many different arrangements of the letters in the word GRADE ? __________

② John has three red cubes, two white cubes, and two green cubes. What is the probability that a blindfolded person would choose a red cube from a bag containing all the cubes? __________

③ If the cube is replaced, what is the probability of drawing a white cube on the second draw? ______

④ If the first cube picked is not replaced, what is the probability of choosing a red cube on the second draw? __________ (Refer to problem #2)

⑤ Beginning with all of the original cubes in the box and not replacing the cube after each draw, what is the probability of making two draws that are green? __________

⑥ In an experiment twenty cards, labeled 1 through 20, are placed in a box. In only one draw, what is the probability of drawing a card naming a multiple of three? __________

⑦ What is the probability of drawing a card naming an even number, in only one draw? __________

⑧ What is the probability that the number on the card is even or a multiple of three? __________

Answers

Probability

① $5! = 120$

② $\frac{3}{7}$

③ $\frac{2}{7}$

④ $\frac{2}{6} = \frac{1}{3}$

⑤ $\frac{2}{7} \times \frac{1}{6} = \frac{1}{21}$

⑥ $\frac{6}{20} = \frac{3}{10}$

⑦ $\frac{1}{2}$

⑧ $\frac{10}{20} + \frac{6}{20} - \frac{3}{20} = \frac{13}{20}$

(6, 12, and 18 are in both sets)

IDEAS

For Teachers

"Word Search"

Objective: To provide reinforcement in addition ("Find the Values" is a helpful prerequisite.)

Levels: 4, 5, 6, 7, 8

Possible answers:
12: *bake, cub, den, leg*
20: *milk, table, fast, low*
28: *house, plant, baseball, coming*

IDEAS

For Teachers

"Out-of-State"

Objective: To provide reinforcement of computational skills and experience with word problems

Levels: 5, 6, 7, 8

Additional information:

It may help if you provide an additional sample problem and show how to use the directory to solve it. After students have made up problems of their own, have them solve the problems that other students have written.

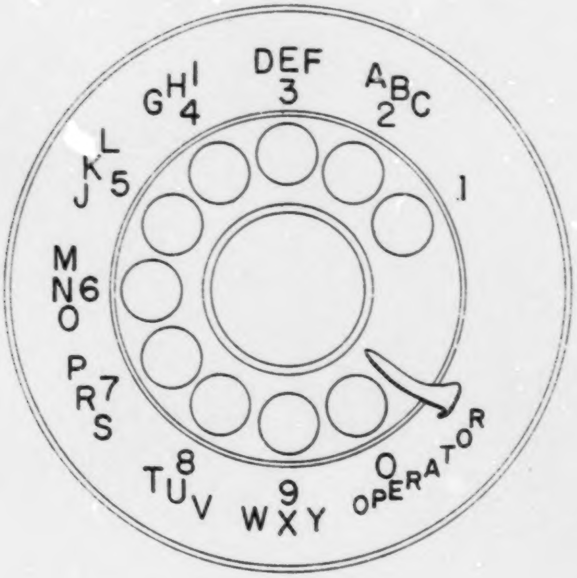
IDEAS

Name ______________________________

WORD SEARCH

Circle one of the numbers below. Then find 5 words worth that number on your telephone dial.

10 20 28


DEF 3
ABC 2
1
GHI 4
JKL 5
MNO 6
PRS 7
TUV 8
WXY 9
0 OPERATOR

____

____

____

____

____

Name ____________________

OUT-OF-STATE

Make up an "out-of-state" telephone problem. Then use the information in the front of the telephone directory to solve it.

Sample: I decided to call my aunt in Memphis, Tennessee. I called at 6 pm on Thursday and spoke for 9 minutes. How much did the call cost?

Problem:

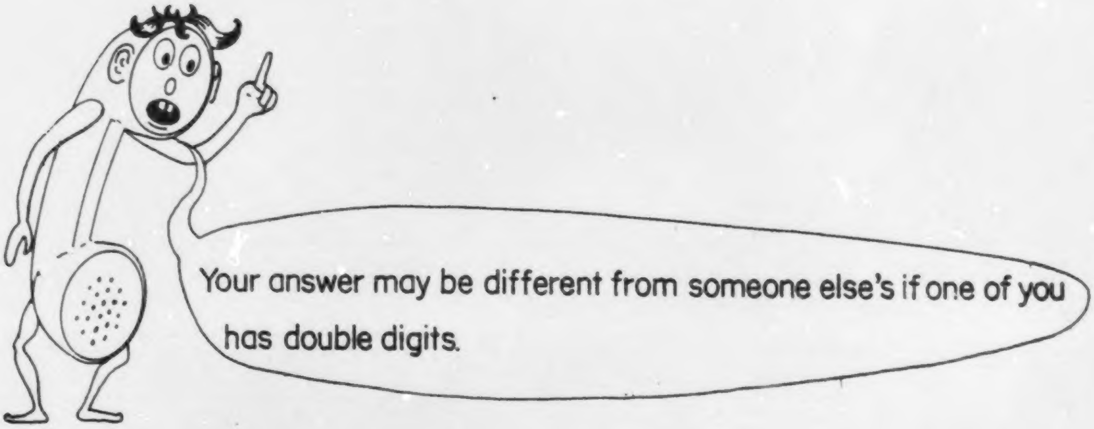
Put your answer on the back.

Name ____________________________

HOW MANY WAYS?

Find all the different ways there are to arrange the last four digits of your telephone number.

Example: 472-3852
3582
8523
and so on


Your answer may be different from someone else's if one of you has double digits.

Maintaining Skills

1. $$\frac{\left(\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{7}{9}\right)}{\frac{1}{2}} \div \frac{\left(\frac{1}{5}+\frac{2}{3}\right)}{\frac{2}{3}} =$$

2. 97432 + 4008 + 63 + 1093 =

3. 56002 − 7681 =

4. 48.024 x .0096 =

5. 749.1609 ÷ .009 =

6. 632.4 + 7421 + 6.094 + .5003 =

7. 9704.6 − 693.8723 =

8. $6\,{}^{2}/_{3} \cdot 7\,{}^{1}/_{4} =$

9. $14\,{}^{2}/_{9} + 5\,{}^{1}/_{2} + 6\,{}^{2}/_{3}$

10. $7\,{}^{1}/_{3} - 3\,{}^{5}/_{6} =$

11. $9\,{}^{1}/_{5} - 2\,{}^{1}/_{3} =$

12. ${}^{2}/_{9}$ = ____________ (percent)

13. 7.4% = ____________ (decimal)

14. .93 = ____________ (percent)

15. ${}^{5}/_{6}$ % = ____________ (fraction)

16. What percent of 21 is 14? ____________________

17. 17 is what percent of 33? ____________________

18. 22% of what number is 54? ____________________

19. 18% of 54 is what number? ____________________

Answers:

1. 28
   39

2. 102, 596

3. 48321

4. .4610304

5. 83240.1

6. 8059.9943

7. 9010.7277

8. $48\frac{1}{3}$

9. $26\frac{7}{18}$

10. $3\frac{1}{2}$

11. $3\frac{33}{35}$

12. $22\frac{2}{9}$ %

13. .074

15. $\frac{1}{120}$

16. $66\frac{2}{3}$ %

17. $51\frac{17}{33}$ %

18. $245\frac{5}{11}$

19. 9.72

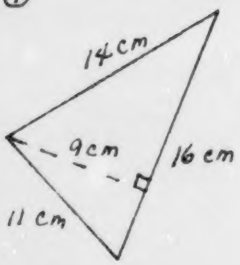
Mathematics Objective

54. Find the perimeter and area of rectangles, parallelograms, triangles and trapezoids.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
perimeter	Perimeter
For teaching suggestions see page 238 8 (Scott Foresman)	For teaching suggestions see page 98 8 (Scott Foresman)
Area	Area
For teaching suggestions see page 218-221 8 (Scott Foresman)	For teaching suggestions see page 154-160 7 (Scott Foresman)
Perimeter	Perimeter of a polygon
For teaching suggestions see page 192-193, 204 8 (Silver Burdett)	For teaching suggestions see page 298-99 7 (Silver Burdett)
Area	Area
For teaching suggestions see page 196-199 8 (Silver Burdett)	For teaching suggestions see page 302-308 7 (Silver Burdett)
	Practice 8 - 83
	In Guide: Area and Perimeter
	MRC: PAWP: CC 39

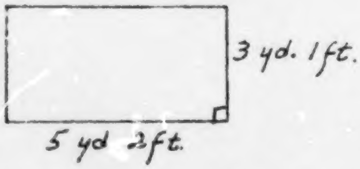
Area and Perimeter

①


14 cm
9 cm
16 cm
11 cm

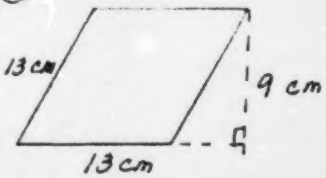
Area = ________
Perimeter = ______

②


3 yd. 1 ft.
5 yd 2 ft.

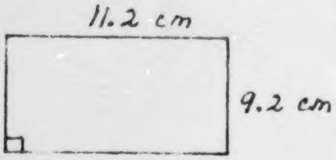
Area = ________
Perimeter = ________

③


13 cm
9 cm
13 cm

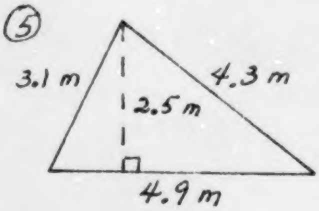
Area = ________
Perimeter = ________

④



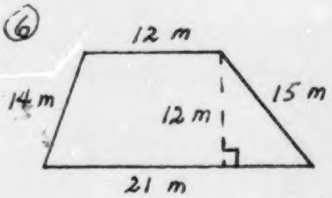
Area = ________
Perimeter = ______

⑤


3.1 m
4.3 m
2.5 m
4.9 m

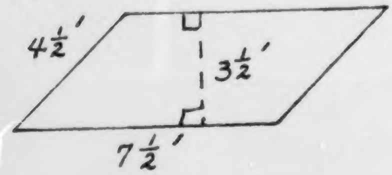
Area = ________
Perimeter = ______

⑥


12 m
14 m
12 m
15 m
21 m

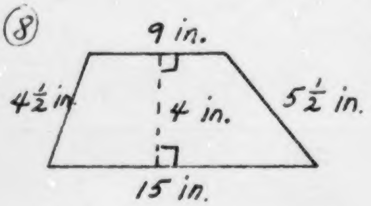
Area = ________
Perimeter = ______

⑦



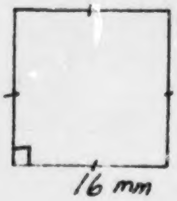
Area = ________
Perimeter = ______

⑧


9 in.
4½ in
4 in.
5½ in.
15 in.

Area = ________
Perimeter = ________

⑨



Area = ________
Perimeter = ______

The Garden Fence

Suppose you are fencing a garden and have only twenty-four meters of fence. You want to have the largest possible area for your garden. In what shape should you place the fence? In this activity you will discover the answer.

1. Cut a piece of string or thread so that when it is tied into a loop, the perimeter is twenty-four units. Use this line to measure:

12 units

2. On the grid below, use your loop to make several possible shapes for the garden fence. Don't use just rectangles or even just polygons. By counting square units, determine the area bounded by each of your shapes and record the results in the table.

   To make counting square units easier, first count whole squares, then look for two or more pieces that together make up approximately a whole square unit. Be sure to mark squares as they are counted.

Shape	Area
#1	
#2	
#3	
#4	

3. If two regions have the same perimeter, do they necessarily have the same area?

Areas of Rectangles

1. In this activity, suppose that the garden has a rectangular shape. On the grid below, draw at least six different rectangles, each with a perimeter of twenty-four units. Be sure to include a square.

2. Determine the area bounded by each rectangle and fill in the table.

3. Which rectangle bounds the largest area?

4. A polygon is called regular if its sides are all of equal length and its angles are all congruent. Is the rectangle that bounds the largest area regular?

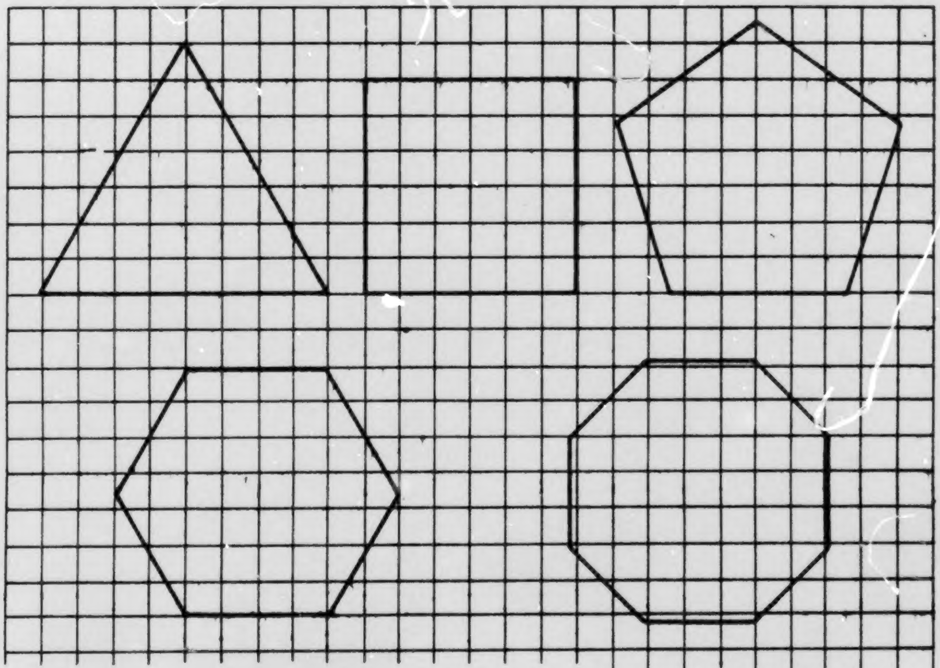
Dimensions of rectangle	Area enclosed

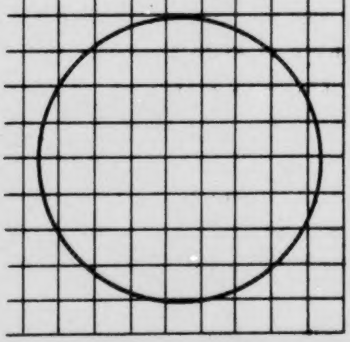
Areas of Regular Polygons

Number of sides	Area
3	
4	
5	
6	
8	

From sheet 2, you found that the rectangle that bounds the greatest area is regular. For a fixed number of sides, the polygon that bounds the largest area is always regular.

1. Each of the regular polygons below has a perimeter of twenty-four units. As accurately as you can, determine the area bounded by each polygon and fill in the table.





2. What happens to the area as the number of sides increases?

3. If a regular polygon had a billion sides, what shape would it resemble?

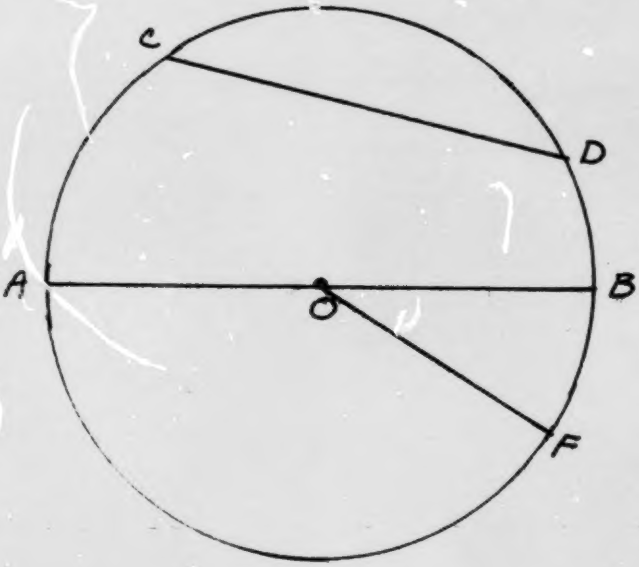
4. Find the area bounded by this circle.

5. Look at the three tables you have completed and at your answer to exercise 4. Which shape should you use for your garden fence?

56M Identify parts of a circle

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
The circle For teaching suggestions see page 204 8 (Scott Foresman) Central angles and arcs For teaching suggestions see page 206 8 (Scott Foresman) The circle For teaching suggestions see page 194 Arc length and area of a sector of a circle For teaching suggestions see page 202 8 (Silver Burdett)	The circle For teaching suggestions see page 132 7 (Scott Foresman) The circle For teaching suggestions see page 290 7 (Silver Burdett) In Guide: Parts of a circle

Parts of a Circle

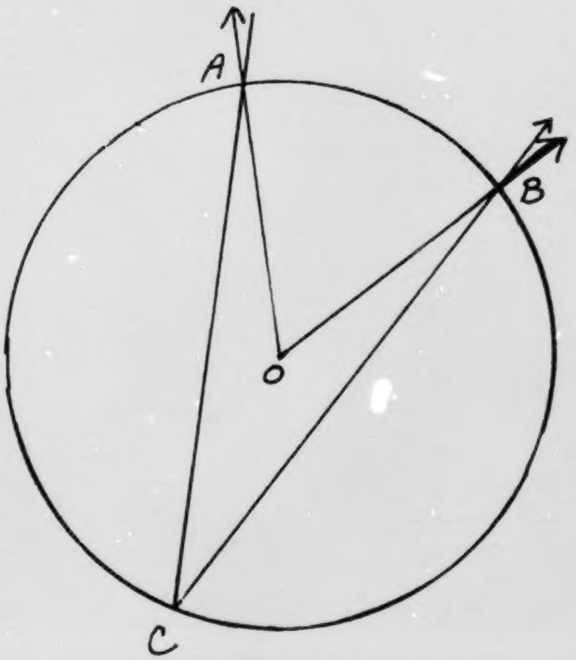


Point O is the center of the circle

A chord is a line segment with end points on the circle. $\overline{CD}$ is a chord. $\overline{AB}$ is a chord.

A chord that contains the center of the circle is a diameter of the circle. $\overline{AB}$ is a diameter.

A radius has its endpoints at the center of the circle and on the circle. $\overline{OF}$ is a radius.



A arc is a part of the circle. $\overset{\frown}{CD}$ is an arc. $\overset{\frown}{CAF}$ is a arc.

A central angle has its vertex at the center of the circle. $\angle AOB$ is a central angle.

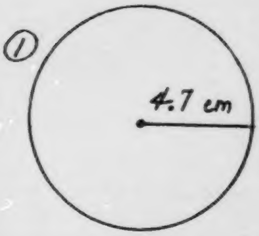
An inscribed angle has its vertex on the circle. $\angle ACB$ is an inscribed angle.

56. Find the circumference of a circle
57. Find the area of a circle

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 226-229 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)	practice 8 - 86, 87 8-H 24
For teaching suggestions see page 194, 200 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)	
There is interesting information on TT on page 195, Teacher's Corner.	Skillmaster 8-46, 47
	MRC: MS- F-42, 43, 44
	Amusements in mathematics in the Development of Geometry Series pp. 55-56 pp. 62

Circumference and Area

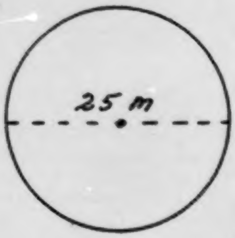
①



Area = ____________________

Circumference = ____________

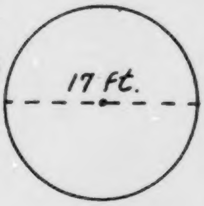
②



Area = ____________________

Circumference = ____________

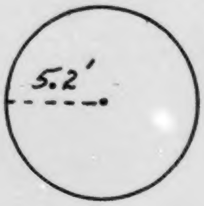
③



Area = ____________________

Circumference = ____________

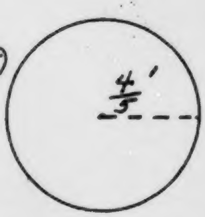
④



Area = ____________________

Circumference = ____________

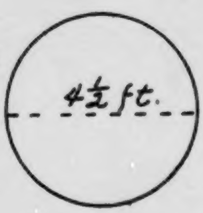
⑤



Area = ____________________

Circumference = ____________

⑥



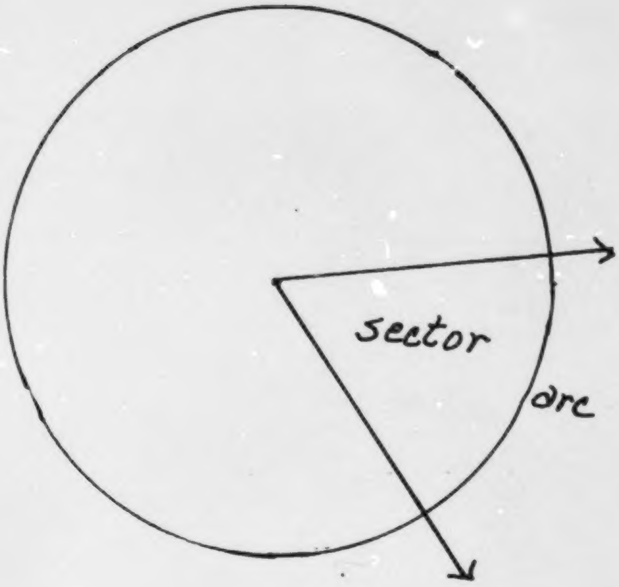
Area = ____________________

Circumference = ____________

57a Determine the measure of angles and areas (sectors and segments contained in a cricle)

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Arc length and area of a sector p. 202 8 (Silver Burdett)	In Guide: Arcs and Sectors

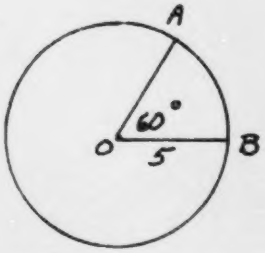
Arcs and Sectors



A central angle determines an arc and a sector.

The sum of all the angles with vertex at the center of the circle measure 360°.

If the central angle measures $n°$, $n°$ is $\frac{n}{360}$ of the entire circle. The length of the arc determined by an $n°$ central angle is $\frac{n}{360}$ of the circumference of the circle.
The area of the sector determined by an $n°$ central angle is $\frac{n}{360}$ of the area of the circle.

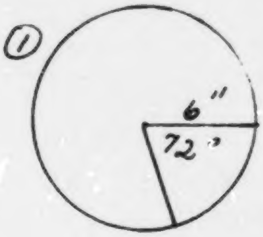


$$\text{length of } \widehat{AB} = \frac{60}{360} \times C$$
$$= \frac{1}{6}(3.14)(10)$$
$$= 5.2\overline{3}$$
$$\approx 5.23 \text{ (nearest hundredth)}$$

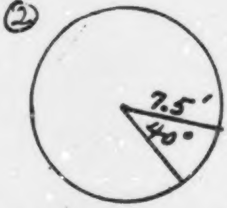
$$\text{Area of sector} = \frac{60}{360} \times A$$
$$= \frac{1}{6}(3.14)(5)^2$$
$$\approx 141.67$$
(nearest hundredth)

Find the length of the arc and the area of the sector. Round your answer to the nearest thousandth.

①



②



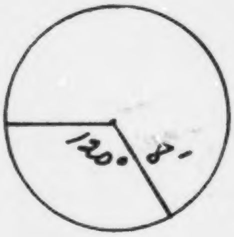
length of arc = ____________

Area of sector = ____________

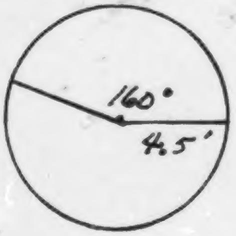
length of arc = ____________

Area of sector = ____________

③



④



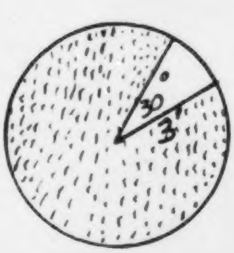
length of arc = ____________

Area of sector = ____________

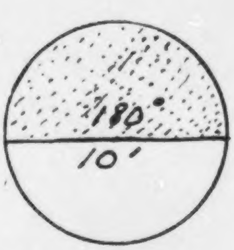
length of arc = ____________

Area of sector = ____________

⑤



⑥



length of arc = ____________
(shaded)

Area of sector = ____________
(shaded)

length of arc = ____________
(shaded)

Area of sector = ____________
(shaded)

Answers

Arcs and sectors

① arc = 7.54°
sector = 22.61 in.^{2}

② arc = 5.23°
sector = 19.63 ft.^{2}

③ arc = 16.75°
sector = 66.99 ft.^{2}

④ arc = 12.56°
sector = 28.26 ft^2

⑤ arc = 103.62°
sector = 25.905 ft^2

⑥ arc = 15.7°
sector = 39.25 ft.^{2}

Mathematics Objective

58. Find the surface area of polyhedrons

Teaching Techniques and Introductory Activities

Surface area of polyhedrons
For teaching suggestions see page 222 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman)

Surface area of polyhedrons
For teaching suggestions see page 324, 326, 328 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)

Supplementary Activities and Resources

Surface Area of a prism and pylamid
p. 176-179
7 (Scott Foresman)

practice 8-24
8 L-47
8 H-22

Surface area of a prism
p. 319-322
7 (Silver Burdett)

In Guide: Total Surface Area of Boxes,
The Mathematics Teacher, Oct. 1972

MRC: Amusements in mathematics in the
development of Geometry
Series: pp. 67

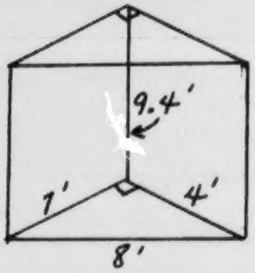
Mathematics Objective

59. Find the volume of a prism or pyramid.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Volume of a prism and pyramid For teaching suggestions see page 224-225 of the Teacher's Edition of *Mathematics Around Us* 8 (Scott Foresman) Volume of a prism and pyramid For teaching suggestions see page 337, 339 of the Teacher's Edition of *Mathematics for Mastery* 8 (Silver Burdett)	Volume of a prism p. 180-181 7 (Scott Foresman) Practice 8-85 8 L-48 H-23 Skillmaster 8-72, 73 Volume of a prism p. 327-329 7 (Silver Burdett) In Guide: Prism Pyramids Surface Area and Volume (prisms) Surface Area and Volume (pyramids) (answers) MRC: Amusements in mathematics in the Development of Geometry Series. pp. 65-66

Surface Area and Volume
(prisms)

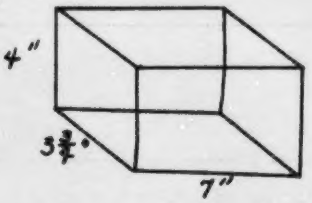
①



Surface Area = ____________

Volume = ________________

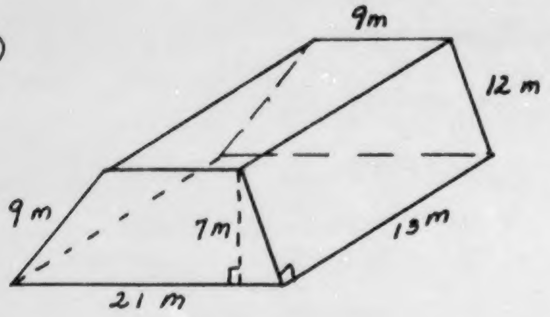
②



Surface Area = ____________

Volume = ________________

③

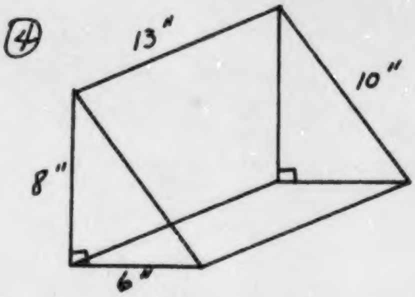


9m
12 m
9 m
7m
13m
21 m

Surface Area = ____________

Volume = ________________

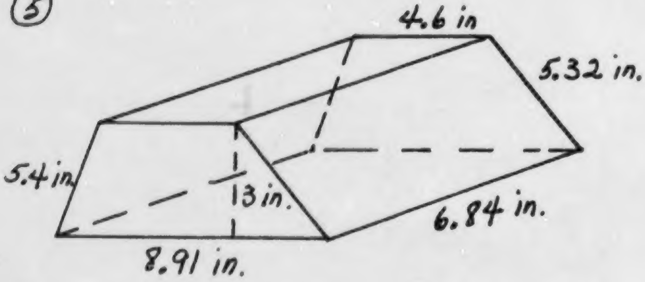
④



Surface Area = ____________

Volume = ________________

⑤



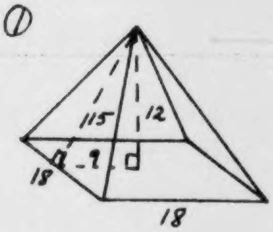
4.6 in
5.32 in.
5.4 in
13 in.
6.84 in.
8.91 in.

Surface Area = ____________

Volume = ________________

Surface Area and Volume
(pyramids)

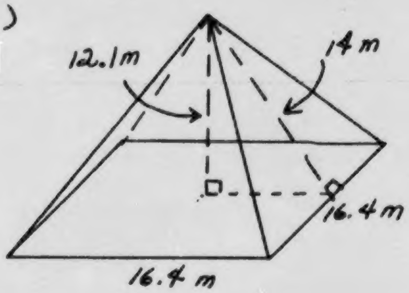
①



Surface Area = ____________

Volume = ______________

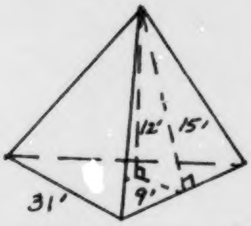
②


12.1m
14 m
16.4m
16.4 m

Surface Area = ____________

Volume = ____________

③

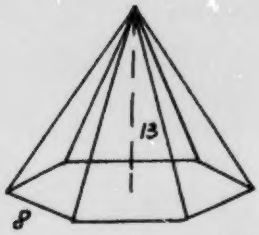


Area of base = 418.5 ft^2

Surface Area = ______________

Volume = ______________

④



slant height = 14.7
Area of base = 45.032

Surface Area = __________

Volume = __________

Answers

Area and perimeter

① $A = 72\ cm^2$
$P = 41\ cm$

② $A = 18\frac{8}{9}\ yd.^2$
$P = 18\ yd.$

③ $A = 117\ cm^2$
$P = 52\ cm$

④ $A = 103.04\ cm^2$
$P = 40.8\ cm$

⑤ $A = 6.125\ m^2$
$P = 12.3\ m$

⑥ $A = 198\ m^2$
$P = 62\ m$

⑦ $A = 26\frac{1}{4}\ ft.^2$
$P = 24\ ft.$

⑧ $A = 48\ in.^2$
$P = 34\ in.$

⑨ $A = 256\ mm^2$
$P = 64\ mm$

Circumference and Area

$\pi \approx 3.14$ $\pi \approx \frac{22}{7}$

① $A = 69.3626\ cm^2$ $(69\frac{149}{350})$
$C = 29.516\ cm$ $(29\frac{19}{35})$

② $A = 490.625\ m^2$ $(491\frac{1}{14})$
$C = 78.5\ m$ $(78\frac{4}{7})$

③ $A = 226.475\ ft.^2$ $(227\frac{1}{14})$
$C = 53.38\ ft.$ $(53\frac{3}{7})$

④ $A = 84.9056\ ft.^2$ $(84\frac{172}{175})$
$C = 32.656\ ft.$ $(32\frac{2}{5})$

⑤ $A = 20.096\ ft^2$ $(2\frac{2}{175}\ ft^2)$
$C = 5.024\ ft$ $(5\frac{1}{35}')$

⑥ $A = 15.89625\ ft^2$ $(15\frac{51}{56})$
$C = 14.13\ ft$ $(14\frac{1}{7})$

Surface Area and Volume (prisms)

① $S.A. = 206.6\ ft.^2$
$V = 131.6\ ft.^3$

② $S.A. = 138.5\ in.^2$
$V = 105\ in.^3$

③ $S.A. = 873\ m^2$
$V = 1365\ m^3$

④ $S.A. = 360\ in.^2$
$V = 312\ in.^3$

⑤ $S.A. = 206.2632\ in.^2$
$V = 138.6126\ in.^3$

Surface Area and Volume (pyramids)

① $S.A. = 864$ sq. units
$V = 1296$ cubic units

② $S.A. = 728.16\ m^2$
$V = 1084.8053\ m^3$

③ $S.A. = 1116\ ft.^2$
$V = 1674\ ft.^3$

④ $S.A = 397.832$ sq. units
$V = 195.13866$ cu. units

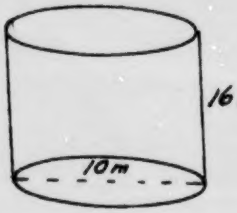
Mathematics Objective

60. Find the volume of a cyliner or a cone
60a. find the surface area of a cylinder or a cone

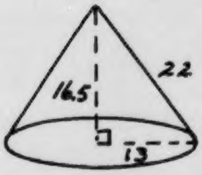
Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Volume For teaching suggestions see page 230, 231 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)	
Surface area For teaching suggestions see page 232 SF	practice 8-88
Volume For teaching suggestions see page S.B. page 340	Skillmaster 8-74 In Guide: Cylinders, Cones Surface Area and Volume (cylinders and cones) answers
Surface area For teaching suggestions see page 330-31	MRC: AMID G S. p. 73, 74 Amusements in Mathematics in the Development of Geometry Series pp. 73-74

Surface Area and Volume
(cylinders and cones)

①



②

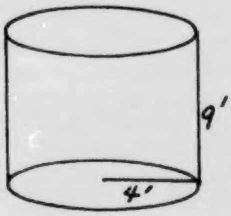


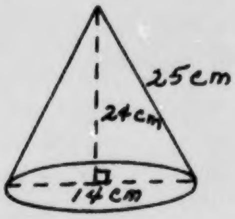
Surface Area = ______________

Volume = ______________

Surface Area = ______________

Volume = ______________



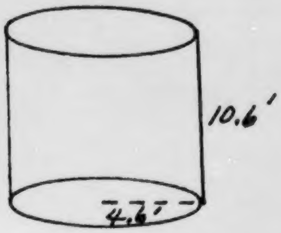

25cm
24cm
14cm

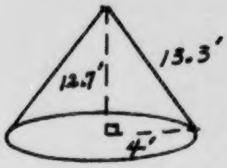
Surface Area = ______________

Volume = ______________

Surface Area = ______________

Volume = ______________





Surface Area = ______________

Volume = ______________

Surface Area = ______________

Volume = ______________

Mathematics Objective

60b. find the surface area and volume of a sphere

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 332, 341 of the Teacher's Edition of *Mathematics for Mastery* (Silver Burdett)	Skillmaster 8 p. 8-75 In Guide: Sphere Surface Area and Volume (spheres) answers MRC: Amusements in Mathematics in the Development of Geometry Series: p. 75

• ACTIVITIES
TOTAL SURFACE AREA OF BOXES

By L. Carey Bolster, Baltimore County Public Schools, Towson, Maryland

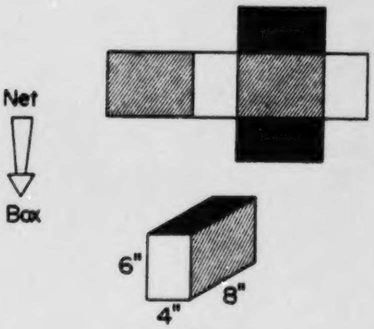
Teacher's Guide

Grade level: 7 and 8

Materials: Colored tagboard; student worksheets

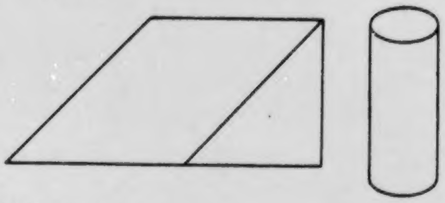
Objectives: Given a figure drawn in perspective, the student should be able to name the dimensions of various faces, compute the area of each face, describe a method for finding the Total Surface Area of a box, and apply this method to solve problems.

Directions: Construct a net of a box using different colored tagboard (for example, the front and back faces—red; the top and bottom—blue; and the left and right—yellow). Assemble the box with tape.



Hold the box in front of the class and have a student sketch the box on the board. Have a student measure the length, width, and height of the box and label the sketch. Students should discuss different methods for finding the Total Surface Area. Open the box and examine the net. List the faces having the same color on the board. Students should notice that they must find the area of each face and then add to find the Total Surface Area.

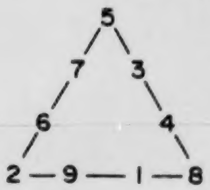
Distribute the worksheets. Check each section before proceeding. After completing the worksheet, some students may only be able to describe a method for finding Total Surface Area, while others are capable of stating the general formula in symbols, that is, $2(lw + wh + lh)$. Both responses should be accepted as being correct. This topic may be extended to find the Total Surface Area of other shapes, such as:



Answers:

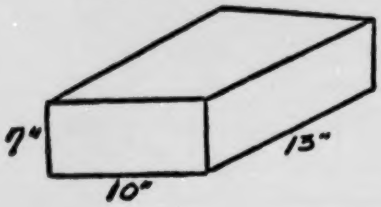
1. 70 sq. in.
2. 70 sq. in.
3. 130 sq. in.
4. 130 sq. in.
5. 91 sq. in.
6. 91 sq. in.
7. 582 sq. in.
8. 288 sq. in.
9. 288 sq. in.
10. 384 sq. in.
11. 384 sq. in.
12. 108 sq. in.
13. 108 sq. in.
14. 1560 sq. in.
15. 1300 sq. in.
16. 384 sq. in.
17. 472 sq. in.
18. Answers will vary

Puzzle:



FINDING TOTAL SURFACE AREA

NAME ______________________________



A. Each face of the figure above is shown below. Label the dimensions and then find the area.

AREA

1. Front Face

70 sq. in

2. Back Face

___ ___ ________

3. Top Face

___ ___ ________

4. Bottom Face

___ ___ ________

5. Right Face

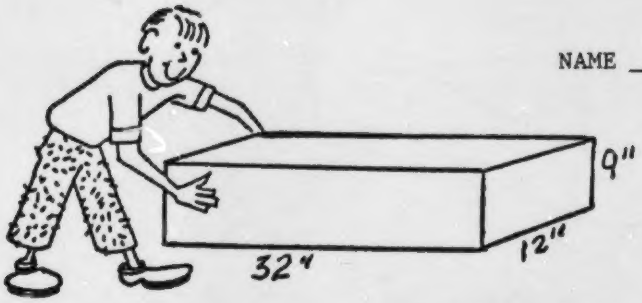
___ ___ ________

6. Left Face

___ ___ ________

The Total Surface Area is the sum of the areas of the faces.

7. Find the Total Surface Area. ________

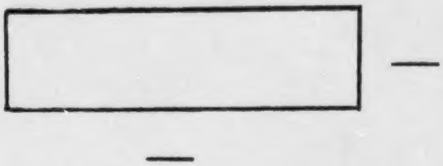


NAME _______________________

B. Some of the faces are not shown below. Name and sketch them. Label all dimensions and find the area of each face.

AREA

8. Front Face

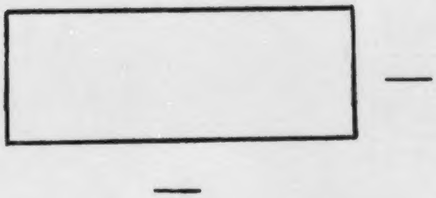


____

9. _______ Face

____

10. Top Face

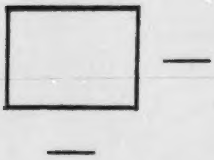


____

11. _______ Face

____

12. Right Face



____

13. _______ Face

____

14. Total Surface Area

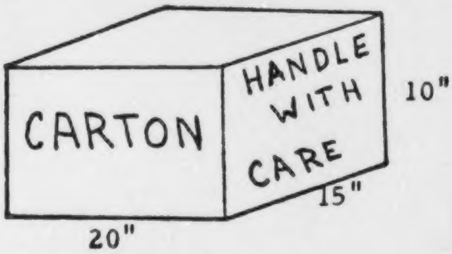
____

NAME ____________________________

C. Find the Total Surface Areas of these boxes.

AREA

15.


CARTON
HANDLE WITH CARE
10"
15"
20"

Front Face ________

Back Face ________

Top Face ________

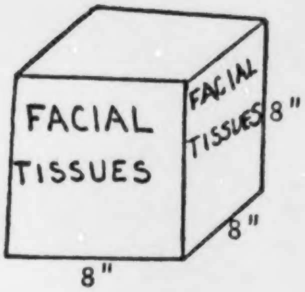
Bottom Face ________

Right Face ________

Left Face ________

Total Surface Area ________

16.


FACIAL TISSUES
FACIAL TISSUES
8"
8"
8"

Front Face ________

________ Face ________

________ Face ________

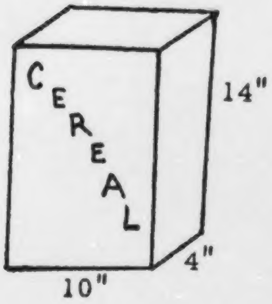
Bottom Face ________

________ Face ________

Left Face ________

Total Surface Area ________

17.


CEREAL
14"
4"
10"

________ Face ________

________ Face ________

________ Face ________

________ Face ________

________ Face ________

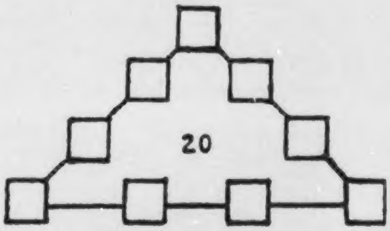
________ Face ________

Total Surface Area ________

18. Describe as many ways as you can for finding the Total Surface Area of a box. Test your methods on the problems above.

Finished? Try this puzzle.

Write each of the nine digits in the squares so that you have a total of 20 on each side.



Prism

	L	W	H	V	S. A.
1	8	10	12		
2	9	16	18		
3	12	18	30		
4	48	60	32		
5	21	9	16		
6	25	25	25		
7	22	23	24		
8	11	12	14		
9	15	16	17		
10	19	20	21		

Pyramids

	S	H_1	H_2	V	S. A.
1	4	4	4.5		
2	5	6	6.5		
3	5.6	10	10.4		
4	6	5	5.8		
5	6.8	9	9.6		
6	7.6	11	11.6		
7	7.8	12	17		
8	8	15	15.5		
9	8.4	7	8.2		
10	13.4	18	19.2		

$V = \frac{s \times s \times H_1}{3}$

$S.A. = (s \times s) + 2\,(s \times H_2)$

Prism

	L	W	H	V	S. A.
1	8	10	12	960	592
2	9	16	18	2592	1188
3	12	18	30	6480	2232
4	48	60	32	92,160	12,672
5	21	9	16	3024	1338
6	25	25	25	15,625	3750
7	22	23	24	12144	3172
8	11	12	14	1848	908
9	15	16	17	4080	1534
10	19	20	21	7980	2398

Pyramids

	S	H_1	H_2	V	S. A.
1	4	4	4.5	$21\ {}^{1}/_{3}$	52
2	5	6	6.5	50	90
3	5.6	10	10.4	108.7	147.84
4	6	5	5.8	60	105.6
5	6.8	9	9.6	138.7	176.8
6	7.6	11	11.6	$211.78\ {}^{2}/_{3}$	234.08
7	7.8	12	17	243.36	326.04
8	8	15	15.5	320	312
9	8.4	7	8.2	164.64	208.32
10	13.4	18	19.2	1077.36	694.12

$V = \frac{s \times s \times H_1}{3}$

$S.A. = (s \times s) + 2\,(s \times H_2)$

Cylinders

	R	D	H	V	S.A.
1	2	4	3		
2	3	6	5		
3	6	12	7		
4	7	14	5		
5	8	16	3		
6	10	20	15		
7	10	20	30		
8	15	30	100		
9	7	14	4		
10	8	16	2		

Cones

	R	D	H	Hs	V	S. A.
1	2	4	7	7.3		
2	3	6	6	6.7		
3	4	8	9	9.8		
4	6	12	15	16.2		
5	7	14	9	11.4		
6	8	16	10	12.8		
7	9	18	11	14.2		
8	2	4	15	15.1		
9	3	6	10	10.4		
10	15	30	30	33.5		

Cylinders

	R	D	H	V	S.A.
1	2	4	3	37.68	62.8
2	3	6	5	141.3	150.72
3	6	12	7	791.28	489.84
4	7	14	5	769.3	572.52
5	8	16	3	602.88	552.64
6	10	20	15	4710	1570
7	10	20	30	8420	2512
8	15	30	100	70,650	10,833
9	7	14	4	615.44	483.56
10	8	16	2	401.92	502.4

Cones

	R	D	H	Hs	V	S. A.
1	2	4	7	7.3	29.3	58.4
2	3	6	6	6.7	56.5	91.4
3	4	8	9	9.8	150.72	173.3
4	6	12	15	16.2	565.2	418.2
5	7	14	9	11.4	461.58	404.43
6	8	16	10	12.8	669.9	522.496
7	9	18	11	14.2	932.58	655.632
8	2	4	15	15.1	62.8	107.388
9	3	6	10	10.4	94.2	126.228
10	15	30	30	33.5	7065	2284.35

Spheres

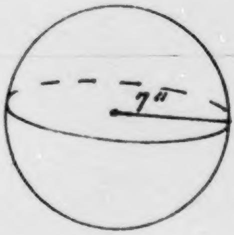
	R.	V.	S.A.
1	3		
2	4		
3	5		
4	6		
5	7		
6	8		
7	9		
8	3.6		
9	4.3		
10	6.8		

Spheres

	R.	V.	S.A.
1	3	113.04	113.04
2	4	267.95	200.96
3	5	523.3	314
4	6	904.32	452.16
5	7	1436.03	615.44
6	8	2143.57	803.84
7	9	3052.08	1017.36
8	3.6	195.33	162.78
9	4.3	309.65	232.23
10	6.8	1316.42	580.77

Surface Area and Volume
(spheres)

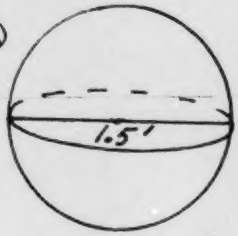
①



Surface Area ________________

Volume ______________________

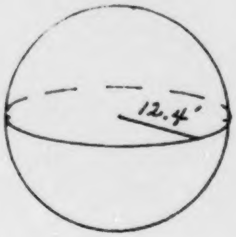
②



Surface Area ______________

Volume __________________

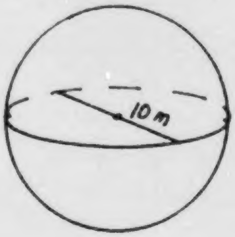
③



Surface Area = ______________

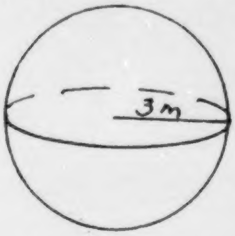
Volume = ____________________

④



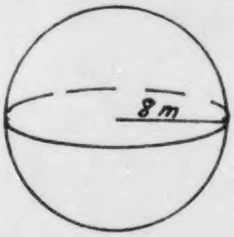
Surface Area ______________

Volume = ____________________



Surface Area = ______________

Volume = ____________________



Surface Area = ______________

Volume = ____________________

Answers

Surface Area and Volume (cylinders and cones)

① S.A = $659.4\ m^2$ (660)
$V = 1256\ m^3$ $(1257\frac{1}{7})$

② S.A. = 1428.7 units^2 (1430)
$V = 2918.63$ units^3 $(2921\frac{2}{7})$

③ S.A. = $326.85\ ft^2$ $(326\frac{6}{7})$
$V = 452.16\ ft^3$ $(452\frac{4}{7})$

④ S.A. = $704\ cm^2$
$V = 1232$ cc

⑤ S.A = $439.0976\ ft^2$ $(439\frac{87}{175})$
$V = 704.28944\ ft^3$ $(704\frac{814}{875})$

⑥ S.A. = $326.56\ ft^2$ $(326\frac{6}{7})$
$V = 212.68266\ ft.^3$ $(212\frac{92}{105})$

Surface Area and Volume (spheres)

① S.A. = $615.44\ in^2$ (616)
$V = 1436.0266\ in^3$ $(1437\frac{1}{3})$

② S.A. = $7.065\ ft^2$ $(7\frac{1}{14})$
$V = 1.76625\ ft^3$ $(1\frac{43}{56})$

③ S.A. = 1931.2256 $(1932\frac{72}{175})$
$V = 7982.399$

(spheres)

④ S.A. = $314\ m^2$ $(314\frac{2}{7})$
$V = 523.\overline{3}\ m^3$ $(523\frac{17}{21})$

⑤ S.A. = $113.04\ m^2$ $(113\frac{1}{7})$
$V = 113.04\ m^3$ $(113\frac{1}{7})$

⑥ S.A. = $803.84\ m^2$ $(804\frac{4}{7})$
$V = 2143.5733\ m^3$ $(2145\frac{11}{21})$

Mathematics Objective

61. Find the square root of a number.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 242, 243 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman) The students might be interested in finding the square root of perfect squares by ordering the squares between squares of multiples of ten. $\sqrt{100} < \sqrt{361} < \sqrt{400}$ $10 < \sqrt{361} < 20$ The square of only two digits en in 1, 1 or 9. Since 361 is closer in vlue to 400 than to 100, the $\sqrt{361} = 19$. $\sqrt{484} = 28$ $\sqrt{400} < \sqrt{484} < \sqrt{900}$ ending in 4 it $20 < \sqrt{484} < 30$ can only be 22 or 28. All of this can be done mentally and the students will enjoy naming square roots of perfect square without referring to a table. In the middle ages the Arabs used this method for finding the square root of a number. formula (1) $\sqrt{a^2 + x} = a + \frac{x}{2a}$ formula (2) $\sqrt{a^2 + x} = a + \frac{x}{2a + 1}$	practice 8-89, 90 8 -L-49

Mathematics Objective

Continued:

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
(1) $\sqrt{83} = \sqrt{81+2} = 9 + \frac{2}{2\cdot 9} = 9 + \frac{2}{18}$ (2) $\sqrt{83} = \sqrt{81+2} = 9 + \frac{2}{2\cdot 9 + 1} = 9 + \frac{2}{19}$ $\sqrt{83}$ is between $9\,{}^{1}/_{9}$ and $9\,{}^{2}/_{19}$ Square roots SB. p. 411, 413, 414	

Algorithm for finding Square Root

1. From the decimal point in either direction group the digits in pairs.

$\sqrt{\overset{\frown}{17}\overset{\frown}{64}.}$ $\sqrt{\overset{\frown}{1}\ \overset{\frown}{56}\ \overset{\frown}{25}.}$

$\sqrt{\overset{\frown}{1}\overset{\frown}{43}.\overset{\frown}{06}}$ $\sqrt{\overset{\frown}{1}.\overset{\frown}{56}\overset{\frown}{24}}$

---

2. Find the greatest integer whose square is equal to or less than the left-hand group. This is the first digit of the square root.

$$\begin{array}{l} \ \ 4 \\ \sqrt{17\ 64} \end{array} \qquad \begin{array}{l} \ \ 1 \\ \sqrt{1\ 56\ 25.} \end{array}$$

$$\begin{array}{l} \ \ 1 \\ \sqrt{1\ 43.06} \end{array} \qquad \begin{array}{l} \ \ 1 \\ \sqrt{1.56\ 24} \end{array}$$

---

③ Subtract the square of this integer from the first group and annex the next group.

$$\begin{array}{r} 4 \\ \sqrt{17\,6\,4} \\ -16 \\ \hline 1\,6\,4 \end{array} \qquad \begin{array}{r} 1 \\ \sqrt{1\ 56\ 25} \\ -1 \\ \hline 0\,56 \end{array}$$

---

④ The next divisor is found by multiplying the part of the root times 20 added to the next digit in the square root.

$$\begin{array}{rr} & 4\ 2 \\ & \sqrt{17\ 64} \\ & -16 \\ 82 \rfloor & 1\,6\,4 \\ & -1\,6\,4 \\ \hline \end{array} \qquad \begin{array}{rr} & 1\ \ 2\ \ 5 \\ & \sqrt{1\ 56\ 25} \\ & -1 \\ \hline 22 \rfloor & 0\,56 \\ & -44 \\ 245 \rfloor & 12\,25 \\ & 12\,25 \\ \hline \end{array}$$

⑤ Each new divisor is found by multiplying the part of the root already found by 20 and adding to it the next part of the root.
(double the part of the root already found and annexing the new digit in the root.)

$$\begin{array}{rr} & 2\ \ \ 5\ \ \ 3 \\ & \sqrt{6\ \ 40\ \ 09} \\ & -4 \\ \hline 4\underline{5} \rfloor & 2\,40 \\ & -2\,25 \\ \hline 50\underline{3} \rfloor & 15\,09 \\ & -15\,09 \\ \hline \end{array}$$

Some squares are non-terminating, non-repeating decimals. (irrational numbers) These must be rounded to a specified place value.

Study the examples:

$3\ 7.\ 1\ 2\ 4 \approx 37.12$

$\sqrt{13\ 78.20\ 00\ 00}$

-9

$6\underline{7}\ |\ 4\ 78$

469

$74\underline{1}\ |\ 9\ 2\ 0$

$7\ 4\ 1$

$742\underline{2}\ |\ 1\ 7\ 9\ 0\ 0$

$1\ 4\ 8\ 4\ 4$

$7424\underline{4}\ |\ 3\ 0\ 5\ 6\ 0\ 0$

$\underline{2\ 9\ 6\ 9\ 7\ 6}$

$.\ 1\ 6\ 0\ 2$

$\sqrt{.02\ 56\ 71\ 00}$

2×1 + next digit 1

$\underline{2\ 6}\ |\ 1\ 56$

2×16 + next digit $1\ 5\ 6$

$\underline{32\ 0}\ |\ 71$

$-\ 0$

2×160 + next

$\underline{3202}\ |\ 7100$

$\underline{6404}$

696

Find the square root:

① $\sqrt{805}$

③ $\sqrt{157.3}$

② $\sqrt{54}$

④ $\sqrt{1.30\ 88}$

Answers

Square Root

① 28.373
② 7.348
③ 12.542
④ 1.144

Pythagorean Property

① 21 m
② 6 ft.
③ 7.616 ft.
④ 9.487 m
⑤ 7.071 m
⑥ 12 cm
⑦ 12 ft.
⑧ 8 ft.
⑨ 7 cm.
⑩ 41 ft.
⑪ 24′
⑫ 37′

Circular Cones

① $h = 24$ in.
S.A. = 703.36 in.^{2} (704)
V = 1230.88 in.^{3} (1232)

② $h_s = 13$ m
S.A. = 942 m^2 ($942\frac{6}{7}$)
V = 753.6 m^3 ($754\frac{2}{7}$)

③ $r = 36$ ft.
S.A. = 13677.84 ft.^{2} ($13690\frac{2}{7}$)
V = 104,448.96 ft.^{3} (104544)

④ $h = 8$ cm
S.A. = 1507.2 cm^2
V = 1884 cm^3
S.A. = ($1508\frac{4}{7}$)
V = ($1885\frac{5}{7}$)

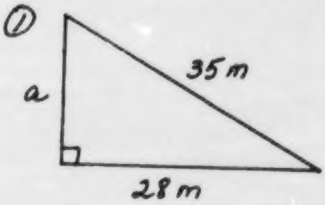
Mathematics Objective

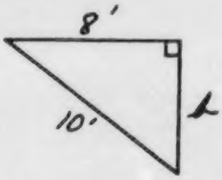
62. Use Pythagorean Property to name the third side of a right triangle, given the measures of two sides.
63. Use Pythagorean Property to solve problems.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
For teaching suggestions see page 246 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)	practice 8-91, 92 8-L 50 8-H 25
For teaching suggestions see page 417, 418, 419 SB.	In Guide: Pythagorean Property Circular Cone (ans)
	MRC: Concept of Geometry: Areas, 16-20
	Amusements in Mathematics in the Development of Geometry Series. p. 31, 34, 35

Pythagorean Property

Find the measure of the missing side: If necessary, round to thousandths.

①


②


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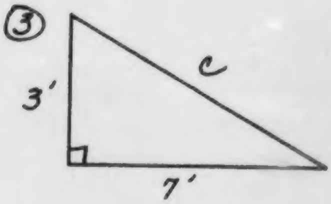
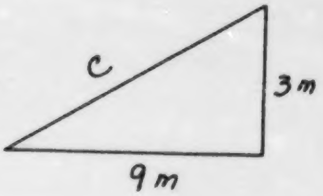
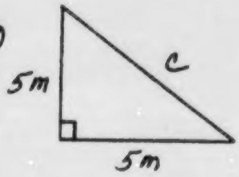


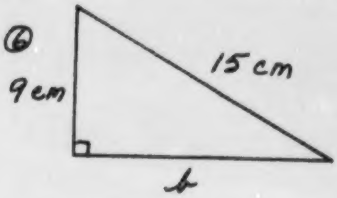
a = ____________

b = ____________

c = ____________

④


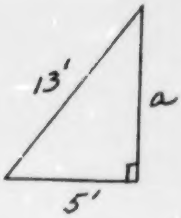
⑤


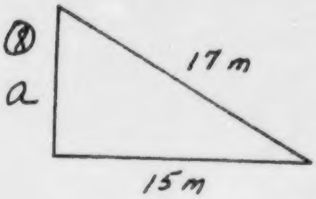
⑥


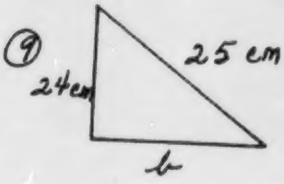
c = ____________

c = ____________

b = ____________

⑦


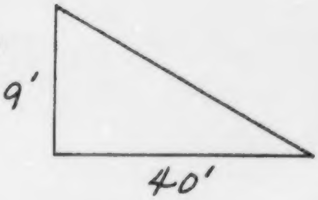
⑧


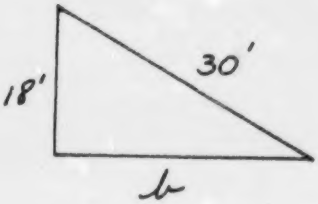
⑨


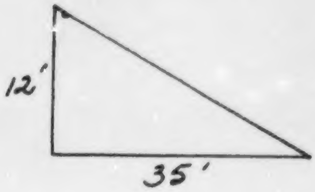
a = ____________

a = ____________

b = ____________

⑩






c = ____________

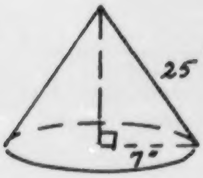
b = ____________

c = ____________

Circular Cones

Use the Pythagorean Property to find missing parts, then solve to find surface area and volume.

①

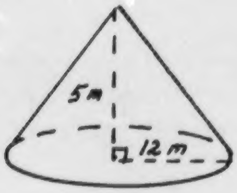


height = ____________________

Surface Area = ______________

Volume =

②

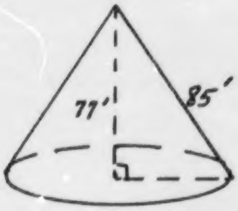


slant height = ______________

Surface Area = ______________

Volume =

③

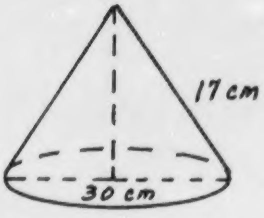


radius = ____________________

Surface Area = ______________

Volume = ____________________

④



height = ____________________

Surface Area = ______________

Volume = ____________________

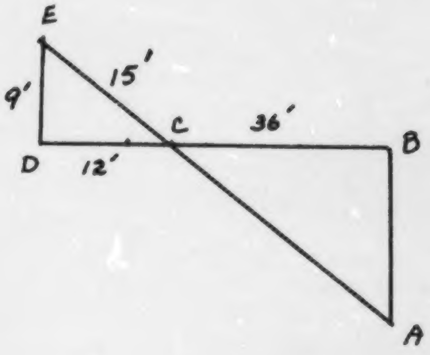
Mathematics Objective

85. recognize similar figures
86. identify corresponding parts of similar figures
87. find missing dimensions of similar triangles using proportion

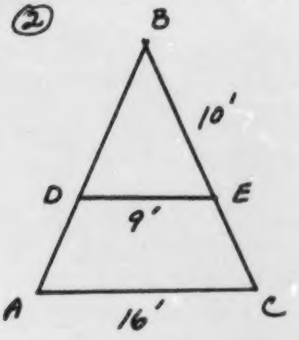
Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Corresponding parts of similar figures For teaching suggestions see page 248, 250 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)	practice 8-93, 94, 97 8-H26, 27
Similar triangles For teaching suggestions see page 70-71 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)	In Guide: Similar Triangles
SB. p. 316, 318, 319	MRC: Amusements in mathematics in the Development of Geometry Series. pp. 28-29 p. 30
p. 318 Teacher's Corner: Students will enjoy this activity.	Concepts of Geometry: Congruence and Similarity
SB. p. 323	

Similar Triangles

①



②



length of line segment AB = ____

length of line segment BC = ____

length of line segment EC = ____

③ A tree 10 feet tall casts an 8 foot shadow. At the same time, a near-by building casts a 32 foot shadow. What is the height of the building? ________.

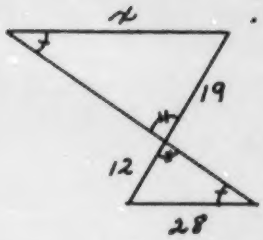
④ Joe is just 5 feet tall. He wants to find the height of a pine tree in front of his house. The shadow cast by the tree was 28 feet long at the same time as her own shadow was 4 feet long. Find the height of the tree. ____

⑤ A tree casts a shadow 125 feet long when a man 6 feet tall casts a shadow 15 feet long. What is the height of the tree? ________

⑥ A tree 30 feet tall casts a shadow 20 feet long, and at the same time a building casts a shadow 50 feet long. What is the height of the building?

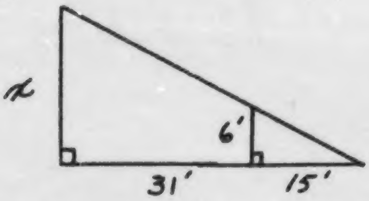
Ans: ① $\overline{AB} = 27'$ ② $\overline{BC} = 17\frac{7}{9}'$ $\overline{EC} = 7\frac{7}{9}'$ ③ 40 ft. ④ 35 ft. ⑤ 50 ft. ⑥ 75 ft.

⑦



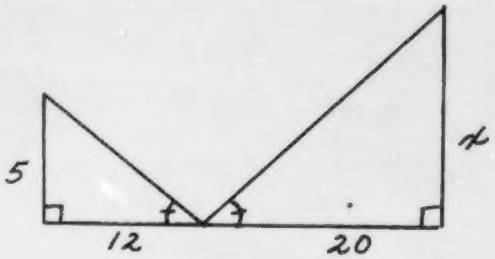
$x =$ ____________

⑧



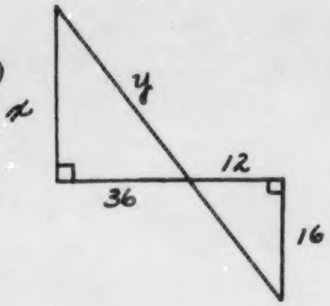
$x =$ ____________

⑨



$x =$ ____________

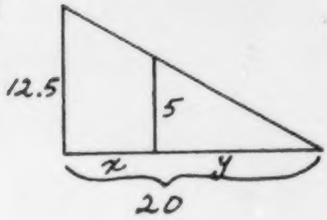
⑩



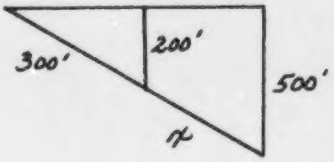
$x =$ ____________

$y =$ ____________

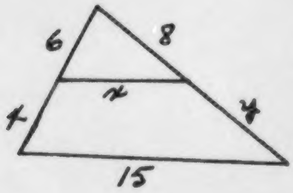
⑪



⑫


300'
200'
500'
x

⑬



$x =$ ____________

$y =$ ____________

$x =$ ____________

$x =$ ____________

$y =$ ____________

Solve the proportions for 'n'.

⑭ $23 : n = 161 : 42$

⑮ $2x : 44 = 16x : n$ $(x \neq 0)$

⑯ $35 : 2x+1 = n : 6x+3$ $(x \neq 0)$

⑰ $2.3 : 3.7 = n : 14.8$

⑱ $4 : n = 12 : 48$

⑲ $3x : 3\frac{4}{5} = 15x : n$ $(x \neq 0)$

Answers

similar Triangles

① 27 ft.
② $\overline{BC} = 17\frac{2}{9}'$
$\overline{EC} = 7\frac{7}{9}'$
③ 40 ft.
④ 35 ft.
⑤ 50 ft.
⑥ 75 ft.
⑦ $44\frac{1}{3}$
⑧ $18\frac{2}{5}$
⑨ $8\frac{1}{3}$
⑩ $48 = x$
$60 = y$
⑪ $x = 8$
$y = 12$
⑫ $x = 450$
⑬ $x = 9$
$y = 5\frac{1}{3}$
⑭ $n = 6$
⑮ $n = 352$
⑯ $n = 105$
⑰ $n = 9.2$
⑱ $n = 16$
⑲ $n = 19$

Mathematics Objective

66a. Find trigonometric ratios in right triangles and solve problems by using trigonometric ratios.

Teaching Techniques and Introductory Activities	Supplementary Activities and Resources
Trignomometric ratios For teaching suggestions see page 273-278 of the Teacher's Edition of *Mathematics Around Us* (Scott Foresman)	practice 8-89, 99, 100 8H-28
Trignonometric ratios For teaching suggestions see page 423 SB.	MRC: Amusements in Mathematics in the development of Geometry Series: pp. 37-38

SUPPLEMENTARY GLOSSARY

Absolute Value: The abolute value of an integer is its distance from zero on the number line.

Acute Angle: An angle with measure less than 90 degrees.

Additive Inverse: Two numbers whose sum is zero are additive inverses of each other. They are also called opposites.

Adjacent Angles: Two angles with a common vertex and a common ray between them.

Alternate Interior Angles: If two parallel lines are cut by a transversal, the angles formed between the parallel lines, one on either side of the transversal are alternate interior angles.

Altitude of a Triangle: A perpedicular segment that extends from one vertex of the triangle to the opposite side.

Angle: The union of two rays with the same end point.

Arc: A part of a circle cut by two radii or a chord.

Area: The measure of a region inside a plane figure.

Associative property of addition: The way in which numbers are grouped does not change the sum.

Associative property of multiplication: The way in which numbers are grouped does not change the product.

Average The sum of a set of numbers divided by the number of members.

Base: The number in the group of a system of numeration. The number on which a place value system is built. Also the number with which an exponent is used. The base is the factor.

Binomial: A phrase that indicates the sum or difference of two numbers.

Bisect: To separate into two congruent parts.

Braces: Symbols used to enclose a set.

Brackets: Symbols of inclusion.

Central angles: An angle whose sides are radii of a circle.

Chord: A line segment whose end points are on a circle. A diameter is a special chord through the center of the circle.

Circle: A closed curve in a plane, formed by points which are the same distance from a given point called the center.

Circumference: The distance around a circle.

Closure: If an operation is performed on any two whole numbers and the result is always a whole number, then the whole numbers are closed under the operation. Similarly for other sets of numbers.

Common denominator: A common multiple of two or more denominators.

Common name: The simplest name for a number.

Commutative property of addition: The order in which two or more numbers are added does not change the sum.

Commutative property of multiplication: The order in which two or more numbers are multiplied does not change the product.

Complementary angles: Two angles are complementary of the sum of their measures is 90 degrees.

Complex fraction: A fraction in which either the numerator of the denominator or both are named by a fraction.

Compound sentences: Sentences using the connective"and" "or" in the former both parts must be true, in the latter only one must be true in order to have true sentences.

Composite number: A whole number greater than zero that has more then two factors.

Concentric circles: Circles in the same plane that have the same center but different radii.

Cone: A space figure formed when a triangle is rotated 360 degrees about one of its legs. The other leg forms a circle which is the base of the cone.

Congruent: Having the same size and shape.

Consecutive angles: Angles in a plane figure that have a common side but not a common vertex.

Coordinates: Numbers associated with points on number line. Ordered pairs of numbers associated with points in a plane.

Coordinate plane: The plane determined by two perpendicular number lines that intersect at the point corresponding to O on each.

Cosine: For a given acute angle of a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse.

Counting numbers: The set of numbers (1, 2, 3, 4, ...)

Cross products: In two fractions or ratios, the products of the numerator in one and the denominator in the other.

Cube: A rectangular prism with all faces congruent square regions.

Curve: In geometry, a path of a point that can be drawn without lifting the pencil.

Cylinder: A space figure formed when a rectangle is rotated 360 degrees about one side. The adjacent sides will be the radii of the circles that form the bases of cylinder.

Decimal: A numeral with a decimal point that names a fractional number whose denominator is a power of ten.

Decimal numeration system: A place-value system based on grouping by tens. The decimal numeration system is sometimes referred to as the base ten system.

Degree: A unit for measuring angles and arcs.

Degree Celsius: The standard unit for measuring temperature in the metric system.

Diagonal: A line segment connecting two non-adjacent vertices of a polygon.

Diameter: A line segment having its end points on a circle and passing through the center of the circle.

Difference: The solution of a subtraction problem.

Discount: An amount deducted from the regular price of an ariticle.

Distributive property: A property that relates addition and multiplication as: $3(6+5)=3(6)+3(5)$.

Dividend: In $35 \div 7$, 35 is the dividend.

Divisor: In $35 \div 7$, 7 is the divisor.

Divisible: When a whole number is divided by another whole number and the remainder is zero, the first is said to be divisible by the second.

Domain: The set of x values of the ordered pairs of a function.

Edge: In a space figure, a line segment where two faces meet.

Element: One of the objects in a set.

Empty set: A set that has no elements. Also called a null set.

End point: The end of a line segment or a ray. The end point of a ray is its vertex.

Equal sets: Those sets which have exactly the same elements.

Equal fractions: Fractions that name the same fractional number.

Equation: A mathematical sentence that states two expresions name the same number. The = sign is used.

Equilateral triangle: A triangle whose three sides are congruent.

Equivalent ratios- Ratios that express the same relationship between ordered pairs of numbers.

Equivalent sets: Two sets that have the same number of elements in them.

Even number: A number with a factor of two.

Event: Any part of the outcomes of an experiment.

Expanded notation: A number written as the sum of the place values of its digits.

Exponent: A small numeral written above and to the right of another numeral to show how many time the base number is used as a factor.

Face: A plane region of a space figure.

Factor: Any one of two or moere numbers whose product is a certain number.

Factorial: The product of a whole number and every whole number between itself and zero.

Finite set: Any set other than an empty set, whose elements can be listed.

Formula: An equation with two or more variables.

Frequency table: In statistics, a listing of the data and how many times each item of data occurred.

Function: A relation in which every ordered number pair has a different first number.

Graph of a number: The point on the number line that corresponds to the number.

Graph of an ordered·pair: The point in corrdinated plane that corresponds to the ordered pair.

Graph of a setence: The graph of the solutions of the sentence.

Gram: A unit of weight in the metric system.

Greatest common factor: The greatest number that is a disor of two or more numbers.

Greatest possible error: One half of the unit of measure used in measuring.

Hexagon: A six-sided polygon.

Hypotenuse: In a right triangle, the side opposite the right angle.

Identity element: For addition, 0; for multiplication, 1. Adding with zero or multipling by one does not change the value of given number.

Improper fraction: Fractions that name numbers greater than or equal to 1.

Inequalities: A mathematical sentence that uses a sign such as $>$. ,$<$,$\leq$, $\geq$, $\neq$.

Infinite set: A set in which the elements can not be counted.

Inscribed angle: An angle whose vertex is on a circle and whose sides are chords of the circle.

Inscribed polygon: A polygon whose sides are chords of a circle, with its vertices on the circle.

Integer: One of the set of numbers that includes all the whole numbers and their opposites.

Interest: Interest is a certain per cent of the amount loaned.

Intersecting lines: Two lines with exactly one point in common.

Intersection of two sets: The set that contains the members that are in both sets.

Inverse/opposite: Subtracting is the opposite of adding. Dividing is the opposite of multiplying.

Irrational number: A number that cannot be represented as a terminating decimal, a terminating decimal, a repeating decimal, or as the quotient of two integers.

Isosceles triangle: A triangle with two congruent sides.

Lateral area: The area of the lateral faces of a space figure.

Lateral faces: The faces of a prism that are not bases.

Least common multiple: The smallest number that is divsible by the given numbers

Legs of a right triangle: The two sides of a right triangle that are opposite the acute angles.

Like fractions: Fractions having the same denominator.

Like terms: Terms that have exactly the same variable or variables and the same powers of these variables.

Line: A straight curve that goes on indefinitely in two directions.

Line segment: A part of a line, consisting of two end points and all the points of the line between them.

Line of symmetry: A line that divides a plne figure into two congruent parts.

Liter: A unit of capacity in the metric system.

Lowest terms: A fraction is in lowest terms if the only common factor of the numerator and the denominator is 1.

Mean: The arithmetic average, the sum of the numbers in a collection of data divided by the number of items.

Median: In a collection of data ranked from least to greatest the median is a number that has as many scores above is as it below it.

Midpoint: The point on a line segment that separates it into two congruent parts.

Meter: A unit of length in the metric system.

Metric system: A system of measurement based on the decimal system.

Mixed number: A numeral with part naming a whole number and part naming a fractional number less than 1.

Mode: The number that occurs most often as a score in a collection of data.

Multiple: The product obtained by multiplying a whole number by another whole number.

Multiplicative inverses: Two number whose product is 1 are multiplicative inerse of each other. They are also called reciprocals.

Numbers of arithmetic: A set of numbers starting at zero and the coordinates of points to the right of zero on the number line.

Obtuse angle: An angle whose measure is greater than 90 and less than 180 degrees. A triangle with an obtuse angle is called an obtuse triangle.

Octagon: An eight sided polygon.

Odd number: A whole number that is not divisible by 2.

Open sentences: An open sentence is a sentence containing a variable that becomes true or false when the variable is replaced by an element from its domain.

Opposite angles: Angles of a polygon that do not have a common side.

Opposites: The opposite of a number is the negative of a number.

Order of operation: To first multiply and divide left to right, then add and subtract left to right after terms in parenthesis have been taken care of.

Ordered pair: A pair of numbers in which the first number is the horizontal distance from zero and the second number is vertical distance from zero.

Origin: On the coordinate plane the point where the two axes intersect.

Outcome: Any one of the possible results of an experiment.

Parallel lines: Lines in the same plane that do not intersect.

Parallelogram: A quadrilateral whose opposite sides are parallel.

Parenthesis: Symbols of inclusion, also used to indicate multiplication.

Pentagon: A five-sided polygon.

Percent: A ratio in which the second term is 100.

Perimeter: The distance around a simple closed curve.

Permutations: The ordered arrangements of a set of objects or numbers in all possible ways.

Perpendicular bisector: A line perpendicular to a segment at the midpoint of the segment.

Perpendicular lines: Two intersecting lines that form right angles at the point of intersection.

Pi: An irrational number. In any circle, the ratio of its circumference to its diameter is equal to pi. (*) 3.14 and 22/7 are approximate values for pi.

Place value: The value given to a digit because of its position in a numeral.

Polygon: A simple closed curve made up of line segments.

Polyhedron; A space figure with polygons for faces.

Power: Another word for exponent.

Precision: A property of a measurement that depends on the unit of measure used. The smaller the unit of measure, the more precise the measurement.

Prime factor: A factor that is a prime number.

Prime factorization: Expressing a composite number as the product of its prime factors.

Prime number: A whole number greater than 1 that has no whole number factors other then 1 and itself.

Prism: A space figure with two parallel faces, called bases, that are congruent. The sides are Parallelogram regions, called lateral faces.

Probability: A number that tells how likely it is that a certain event will happen. It is the number of favorable outcomes divided by the number of possible outcomes.

Product: The solution to a multiplication problem.

Proportion: A sentence that two ratios are equivalent. The numbers in a proportion are called terms.

Protractor: An instrument used to measure angles.

Pyramid: A polyhedron with one base and triangular faces that have a common vertex.

Pythagorean rule: For any right triangle, $a^2+b^2=c^2$, where a and b represent the measures of the legs and c represents the measure of the hypotenuse.

Quadrant: One of the four parts into which a plane is divided by two perpendicular lines.

Quadrilateral: A polygon with four sides.

Quantity: As a whole, the terms inside parenthesis or brackets.

Radius: A segment whose end points are the center of a circle and a point on the circle.

Range: The difference between the greatest and least number in a collection of data.

Range: All the y values of a group of ordered pairs.

Ratio: A pair of numbers that describes a rate or compares two quantities.

Rational numbers: A number that can be expressed as the quotient of two integers, with the divisor not zero. A number that can be named by a terminating decimal or a repeating decimal.

Ray: A part of a line that has one end point and extends endlessly.

Real number: A rational or an irrational number.

Reciprocals: Two numbers whose product is 1.

Region: A simple closed curve together with its interior.

Regular polygon: A polygon that has all its sides congruent and all its angles congruent.

Relation: A collection of ordered pairs of numbers.

Repeating decimal: A decimal in which a digit or a group of digits repeats indefinitely.

Repetend: The part of a repeating decimal that keeps repeating.

Replacement set: The set from which the solution of an equation may be chosen.

Rhombus: A parallelogram whose sides are congruent.

Right angle: An angle whose measure is 90 degrees, formed by perpendicular lines.

Scalene triangle: A triangle with sides of different lengths.

Scientific notation: Two factors are used to express a number in scientific notation. One factor is equal to or greater than 1 but less than 10. The other factor is a power of ten expressed in exponent form.

Sector: Part of a circular region formed by two radii and an arc.

Segment: A part of a line that has two endpoints

Semi-circles: An arc that is one half of a circle.

Sequence: A collection of numbers arranged in order.

Set: A collections of objects that are called elements.

Significant digits: In a measurement, the significant digits are only those that tell the number of times the unit of measure is used.

Similar figures: Figures having the same shape but not necessarily the same size.

Simple closed figure: A figure that can be drawn by beginning at any point and ending at the same point without retracing any other point.

Sine: In a right triangle, the sine of an acute angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.

Skew lines: Two lines not in the same plane that do not meet.

Solution: The number whose name makes the sentence true.

Space figure: A geometric figure whose points lie in more than one plane.

Sphere: A closed surface formed by points the same distance from a fixed point called the center.

Square: A 4 sided polygon with all sides the same length and with four right angles.

Square root: The square root of a number, a, is the number whose square is a.

Subset: A set whose members are all members of another set.

Surface area: The sum of the areas of the faces of a space figure.

Sum: The solution to an addition problem.

Supplementary angles: Two angles are supplementary if the sum of their measure is 180 degrees.

Tangent: A line that touches a circle at only 1 point.

Tangent: In a right triangle, the quotient of the measure of the side opposite the angle divided by the measure of the side adjacent to the angle.

Terms: Those parts of a phrase or sentence separated by addition or subtraction symbols.

Transversal: A line that intersects two or more lines.

Trapezoid: A quadrilateral having exactly one pair of parallel sides.

Triangle: A polygon with three sides.

Union of two sets: When the elements of the sets are joined into one set.

Unlike fractions: Fractions having different denominators.

Unlike terms: When the variable or variables of the terms are not exactly the same, or have different exponents.

Variable: The symbol that takes the place of a numeral in an open sentence.

Vertex: The point of intersection of two sides of a polygon.

Vertical angles: When two lines intersect each pair of nonadjacent angles formed is a pair of vertical angles.

Volume: The measure of the space bounded by a closed space figure.

Whole numbers: The set of counting numbers with zero added.

X axis: The horizontal axis in the coordinate plane.

Y axis: The vertical axis in the coordinate plane.

Zero property: Any number multiplied by zero is equal to zero. Any number added to zero equals that number.